# Second-Order Statistical Discrimination*

Tilman Klumpp† Emory University

Xuejuan Su‡ Emory University

March 2012

**Abstract**

The low representation of female workers in elite jobs is sometimes attributed to a tail effect: If the human capital distribution exhibits less variation among females than among males, then even with comparable average human capital there will be fewer females in the right tail than males. This paper offers an explanation for why the human capital distribution might have this property. We show that the belief that the female human capital distribution has a lower variance than the male distribution can be self-fulfilling, in that it provides individuals with incentives to invest in human capital such that the resulting distribution exhibits exactly this characteristic. If this happens, fewer females are employed in high-end jobs (a "glass ceiling" effect). The average productivity of female workers may at the same time be higher than that of male workers.

**Keywords:** Statistical discrimination, aptitude distribution, gender differences, "glass ceilings."

**JEL codes:** J71, J78.

*We thank Maria Arbatskaya, Paul Peccorino, Paul Rubin, Akram Temimi, the editor Hanming Fang, and two anonymous referees for helpful comments.

†Department of Economics, Emory University, Rich Building 316, 1602 Fishburne Dr., Atlanta, GA 30322. E-mail: tklumpp@emory.edu.

‡Department of Economics, Emory University, Rich Building 322B, 1602 Fishburne Dr., Atlanta, GA 30322. E-mail: xuejuan.su@emory.edu

## 1 Introduction

Job applicants may be treated differently by employers based on their race, gender, or other group membership—even if they are similar in characteristics that are more directly related to their skills, such as their grades. This behavior is rational for employers if an individual's true qualification for a job cannot be perfectly observed *and* groups differ in the average qualification of their members. In this situation, group membership conveys statistical information about an individual's qualifications, and economists use the term *statistical discrimination* to describe the resulting differential treatment of individuals based on the group they belong to. This mechanism, first described in Phelps (1972), assumes that inter-group differences in average qualifications are exogenous. Arrow (1973), and later Coate and Loury (1993), demonstrate that inter-group differences which cause statistical discrimination can also be the result of statistical discrimination: Two a priori identical groups can end up with different average skills if the prospect of discrimination in the labor market discourages one group from investing in their skills. In other words, differences in average qualifications across groups can become self-fulfilling expectations—an idea which has had a profound impact on economists' understanding of labor market inequalities.[1]

In this paper, we extend the self-fulfilling expectations model of statistical discrimination from the first to the second moment of the skill distribution. We show that the same mechanism which can explain differences in means across populations can also explain differences in variance—a phenomenon we call *second-order statistical discrimination.* To motivate this exercise, consider the question why fewer women than men are employed in high-end jobs in business, science, or engineering. One hypothesis is that women are less able than men in these fields, on average. A second hypothesis holds that men and women are similarly able on average, but that the distribution of ability has a higher variance among men than among women. Fewer women than men are then located in the right tail of the ability distribution, which is the relevant region for high-end jobs. The dispersion hypothesis was famously put into the spotlight when former Harvard president Larry Summers made the following remarks at an NBER conference (Summers 2005):

> "It does appear that on many different human attributes [...] there is relatively clear evidence that, whatever the difference in means, which can be debated, there is a difference in the variability of a male and a female population. [...] If one is talking about people who are three-and-a-half,

[1] Most importantly, in order to reduce labor market inequalities arising from (self-fulfilling) statistical discrimination, different policies are required than if discrimination is merely taste-based. While free labor markets tend to self-correct toward a non-discriminatory state in the latter case (see Becker 1953), government intervention such as affirmative action policies can be required in case of statistical discrimination. A survey of the literature of statistical discrimination and policy responses to it, see Fang and Moro (2011).

> four standard deviations above the mean, even small differences in standard deviation will translate into very large differences in the available pool substantially out."

While Summers was criticized for these comments by some colleagues, the claim that a more variable aptitude distribution exhibits more mass in the right tail is intuitive. However, it raises the question: *Why* would this distribution be more variable among men than among women? The notion of second-order statistical discrimination presented in this paper offers a possible explanation.

Our model can be summarized as follows. An individual's aptitude is the product of both his or her innate talent, as well as a costly personal investment in human capital. Neither an individual's innate talent, nor the investment, nor the resulting final aptitude are directly observable. Suppose now that innate talent follows the same distribution in both men and women, but that the individuals' investment decisions result in a final aptitude distribution that has a higher variance among men than among women. If employers observe an imperfect signal of each individual's aptitude, then a low signal from a male applicant must indicate relatively low aptitude, compared to the same signal from a female applicant. Similarly, a high signal from a male applicant indicates high aptitude, compared to the same signal from a female applicant. In assigning workers to jobs, employers will therefore discriminate against men with relatively low signals, and at the same time against women with relatively high signals. The first effect results in a "sticky floor" for men, and the second in a "glass ceiling" for women. We show that this pattern of discrimination discourages women of high innate ability, as well as men of low innate ability, from investing into their human capital. The resulting human capital distribution is then more compressed among women than among men, as was assumed initially, yielding an overall equilibrium.[2]

An important implication of this mechanism is that statistical discrimination can have a non-uniform impact on different individuals in the same group. While conventional, first-order statistical discrimination has a uniform impact on all members of a group, second-order statistical discrimination has differential impact on different members of the same group. In other words, if two individuals from two groups happen to have the same noisy signal, under first-order statistical discrimination the individual from the advantaged (i.e., higher mean ability) group would *always* be favored compared to the individual from the disadvantaged (i.e., lower mean ability) group, regardless of the signal level. On the other hand, under second-order statistical discrimination, a member of the high-variance group could be favored *or* discriminated against, relative to a member of the low-variance group, depending on the value of the signal. Thus, comparing average

[2]At the same time, the mean aptitude of the low-variance group may be below, equal to, or above the mean aptitude of the high-variance group. We also remark that, for the described mechanism to work, individuals do not necessarily have to make their investment decisions themselves. The same mechanism would also work if parents made these decisions, as long as the parents' utility depends on their children's labor market success.

outcomes across groups may mask potentially severe discrimination problems, in that discrimination at certain quantiles of the ability distribution is offset by discrimination in the opposite direction at other quantiles of the distribution.

For example, in 2010 women accounted for 51.5% of employees in management, professional, and related occupations. This ratio suggests that women are not under-represented in these job sectors—if anything, women are slightly over-represented, since they account for only 47.2% of all employed persons.[3] However, in the same year women accounted for only 3% of Fortune 500 CEOs.[4] Looking at average employment outcomes for women therefore obscures the severe employment disparities that exist at the far right end of the outcome distribution. Of course, these numbers are not, in and by themselves, evidence of discrimination against women in obtaining top management positions. But they do suggest that, whatever mechanism is causing these employment disparities, the first moment of the outcome distribution does not convey all relevant information contained in this distribution. In the context of statistical discrimination as one possible mechanism, this means that comparing group-wide average labor market outcomes does not allow detection of more subtle forms of discrimination. Our paper addresses this issue, by developing a model in which statistical discrimination has a differential impact on individuals in the same group, depending on their initial location in the ability spectrum.

The remainder of the paper is organized as follows. In Section2, we review existing research concerned with second-moment differences in the human capital distribution across groups. In Section 3, we introduce our model and show that a non-discriminatory equilibrium always exists. In Section 4, we derive conditions for second-order statistical discrimination and construct one such equilibrium for normally distributed signals. Throughout this analysis, we neglect from wage setting and interpret a job as a "prize" whose value depends only on the type of job an individual is employed in. In Section 5, we discuss the extent to which our results are robust when wages are determined endogenously. Section 6 concludes with a few remarks on policy implications. An Appendix contains all proofs.

## 2 Related Literature

The statistical discrimination literature contains models in which the *conditional* signals of individual qualifications have a higher variance for one group than for another (e.g., Phelps 1972; Aigner and Cain 1977; Lundberg and Startz 1983). In this case, employers place relatively more weight on group averages, and relatively less weight on individual signals, when forming expectations about the qualifications of applicants from

[3]Source: 2010 Current Population Survey (www.bls.gov/cps/cpsaat11.pdf).

[4]Source: USA Today (www.usatoday.com/money/companies/management/story/2011-10-26/women-ceos-fortune-500-companies/50933224/1).

the high-noise group. This, in turn, diminishes the expected return on skill investments for members of the high-noise group, and thus decrease the average skill level in this group. These effects are *not* part of the mechanism we are describing in this paper. We are concerned with second-moment differences in the distribution of actual qualifications, and thus the distribution of unconditional signals, instead of the distribution of signals conditional on qualifications. In order to generate such second-moment differences in equilibrium, our model—like every model of statistical discrimination—makes the assumption that individual qualifications can be observed only imperfectly. However, conditional on qualification, the distribution of signals is identical across all groups in the population. Thus, except for a gender label ("male" and "female"), there are no ex-ante differences across groups in our model.

A model with ex-ante identical groups is developed in Moro (2003). He shows that, when individuals are heterogeneous in their endowed ability and endowed ability is negatively correlated with investment cost in human capital, a statistical discrimination equilibrium exists in which a larger fraction of individuals invest in human capital in one group than in the other. Furthermore, as is the case here, the group that invests more also has a higher variance in its human capital distribution. However, there is an important difference between our model and that in Moro (2003). In the latter, workers are assigned to binary tasks ("simple" and "complex"). Together with the negative relationship between endowed ability and investment cost, this implies that high-ability individuals are always more likely to invest than low-ability individuals, regardless of their group label. Thus, while statistical discrimination disadvantages one group, its impact on investment incentives and outcomes on the disadvantaged group is uniform in the sense that *every* member of the disadvantaged group suffers from statistical discrimination. In contrast, our model has a three-task setup, and—depending on the anticipated probabilities of being assigned to these tasks—it is no longer the case that high-ability individuals are always more likely to invest than low-ability individuals. While high ability individuals in one group may be deterred from investing, low ability individuals may be encouraged, and the opposite may hold in the other group. Statistical discrimination now has a differential impact on high-ability and low-ability individuals in the same group. In fact, as was discussed in the Introduction, a "disadvantaged" group may not exist in our model, as each group may contain some disadvantaged individuals at different ends of the ability spectrum.[5]

[5] A three-task setup is studied in Bjerk (2008), who develops a career ladder model in which promotion of female workers from low-level jobs to mid-level jobs takes longer, on average, than for male workers. Delaying promotion is a form of statistical discrimination that arises from exogenous differences in the precision of skill signals across groups, or the frequency with which signals are observed. There is no discrimination in promotion to top-level jobs, however. Bjerk (2008) shows that a glass ceiling can still arise, due to statistical discrimination earlier in a worker's career. We explore a much different mechanism: Our model is static, and workers are assigned to one of the three tasks without the possibility of later advancement. We then generate a self-fulfilling statistical discrimination equilibrium at the initial (and only) job assignment stage, which requires no ex-ante differences between men and women.

Empirical evidence that comparing mean labor market outcomes across groups can mask more subtle effects occurring only at certain quantiles in the ability distribution is provided in Bjerk (2007). Using the Armed Forces Qualification Test (AFQT) score as a measure for workers' pre-market skills, Bjerk (2007) finds significant differences in skill-sorting thresholds across racial groups: While black workers are less likely than white workers to be employed in white-collar jobs, conditional on AFQT scores blacks are *more* likely to work in white-collar jobs. In particular, Bjerk (2007) finds that the thresholds to sort black workers into white-collar job sectors are significantly lower than those for white workers. While this pattern may have many explanations, it is consistent with our theory that statistical discrimination can have a differential impact on high-ability and low-ability individuals in the same group. If employers believe that the black group has a higher skill variance than the white group, then, conditional on a high AFQT score, a black worker has a higher expected productivity than a white worker with the same score, and is hence favored by potential employers in white-collar job sectors. Thus, individual black workers on the high end of the ability distribution can benefit from statistical discrimination, despite the fact that blacks have a lower average skill level than whites.

Finally, there exists another, biological explanation of second-moment differences in the male and female human capital distribution (see Rubin and Paul 1979; Browne 1998; Browne 2006, and references therein). According to this explanation, males are evolutionarily conditioned to be more risk seeking than females, resulting in a more variable distribution of traits among males, compared to females. The basic premise of the biological hypothesis is that male inputs in reproduction are rationed by female inputs, with some males having access to multiple mates and others being excluded from reproductive opportunities. Excluded males have an incentive to engage in risky activities with the potential to advance their social status in order to gain access to females.[6] Activities which are risky in terms of their direct material consequences thus have little downside risk in terms of their evolutionary success. No such risk-taking incentive exists for females, who comprise the rationing side in the mating game. This biological explanation and the social hypothesis presented here are not mutually exclusive, and we do not take a position as to which of them explains a larger share of any gender differences in occupation or aptitude.[7] Our aim is simply to demonstrate that the same well-known social mechanism that can be used to explain differences in means across populations—namely, statistical discrimination with self-fulfilling expectations—can also explain differences in variance.

[6] The relationship between status seeking and risk seeking is explored also in Becker et al. (2005) and Ray and Robson (2011).

[7] We do contend, however, that in the case of characteristics such as technical skills or mathematical ability—which depend not only on innate talent but also clearly on personal learning investments—a purely biological explanation is unlikely to provide a sufficient account of all gender differences in these variables.

## 3 The Model

The population consists of two groups, male and female, denoted $g \in \{m, f\}$. Each group comprises a continuum of measure 1, so the total population has measure 2. Group membership is publicly observable and has no economic significance ex-ante. That is, all assumptions we make in the model apply equally to both groups.

### 3.1 Human capital production

Each individual is endowed with an initial ability, which can be either $a$ or $b$, with $b > a$. The fraction of males and females with ability $a$ is $\lambda \in (0, 1)$. After learning their ability, individuals decide to invest either low effort $\underline{e}$ or high effort $\overline{e}$ in human capital production. Individuals who spend $\underline{e}$ obtain human capital equal to their initial ability. On the other hand, ability $a$-individuals who invest $\overline{e}$ obtain human capital $A > a$, and ability $b$-individuals who invest $\overline{e}$ obtain human capital $B > b$. The cost of effort $\underline{e}$ is zero, and the cost of effort $\overline{e}$ is $c > 0$ regardless of ability. An individual's initial ability, effort choice, and final human capital are private information.

The set of possible human capital levels is denoted $K \equiv \{a, A, b, B\}$. We make two additional assumptions. First, $b > A$: High effort from a low-ability individual is insufficient to overcome a high-ability individual's initial advantage. Second, $B - b > A - a$: Effort has a larger effect on the human capital of high-ability individuals than low-ability individuals.

A ***strategy*** for an individual of gender $g \in \{m, f\}$ is a mapping $\sigma_g : \{a, b\} \to [0, 1]$, describing the probability of choosing the high effort level $\overline{e}$ as a function of gender and initial ability. We call the pair $(g, q) \in \{m, f\} \times K$ an individual's type. Given strategy $\sigma = (\sigma_m, \sigma_f)$, the measure of individuals of type $(g, q)$ is given by

$$z_g(q) = \begin{cases} \lambda(1 - \sigma_g(a)) & \text{if } q = a, \\ \lambda\sigma_g(a) & \text{if } q = A, \\ (1 - \lambda)(1 - \sigma_g(b)) & \text{if } q = b, \\ (1 - \lambda)\sigma_g(b) & \text{if } q = B. \end{cases} \tag{1}$$

### 3.2 Testing

After individuals have made their effort choices, a publicly observable noisy signal $\theta \in (\underline{\theta}, \overline{\theta}) \subseteq \mathbb{R}$ of an individual's human capital $q \in K$ is generated. The terminology we adopt here is to call $\theta$ a "test score." That is, we think of $\theta$ as the result of an examination all individuals must take after they have made their effort choices and have acquired their human capital.

We make the following assumptions. Conditional on an individual's human capital $q$, $\theta$ has continuous, positive density $f(\theta|q)$ for all $\theta \in (\underline{\theta}, \overline{\theta})$. The corresponding cumulative density is $F(\theta|q)$. To provide information about an individual's human capital, $f$ satisfies the ***monotone likelihood ratio property***:

$$\forall q > q' : \quad \theta > \theta' \;\Rightarrow\; \frac{f(\theta|q)}{f(\theta|q')} \geq \frac{f(\theta'|q)}{f(\theta'|q')} \;\; (> \text{ for some } \theta, \theta'). \tag{MLRP}$$

This property states that, as an individual's human capital increases, high test scores become more likely *relative* to low test scores. Furthermore, $f$ satisfies the following ***separation property***:

$$\lim_{\theta \to \underline{\theta}} \frac{f(\theta|B)}{f(\theta|a)} \;<\; \frac{\lambda}{1-\lambda}\frac{A-a}{B-b} \;<\; \lim_{\theta \to \overline{\theta}} \frac{f(\theta|B)}{f(\theta|a)}. \tag{SEP}$$

This property states that test scores in the left (right) tail of the distribution are sufficiently likely (unlikely) to have come from individuals with the lowest human capital $a$, *relative* to individuals with highest human capital $B$.

The pair $(g, \theta) \in \{m, f\} \times \mathbb{R}$ will be called an individual's ***public type***. Given the human capital distribution $z$ defined in (1), we can express the density of individuals of public type $(g, \theta)$ in the population as $\widetilde{z}_g(\theta) = \sum_{q \in K} z_g(q) f(\theta|q)$.

### 3.3 Job market

There are three different types of jobs in this economy: "Simple jobs" (level 0 jobs), "clerical jobs" (level 1 jobs), and "elite jobs" (level 2 jobs). The measure of available level-$i$ jobs is $\beta_i > 0$ with $\beta_0 + \beta_1 + \beta_2 = 2$ (i.e., every individual can be employed). For the time being we follow Coate and Loury (1993) and assume fixed wages. That is, we assume that individuals attach value $V_i$ to employment in job level $i$, with $\omega_0 < \omega_1 < \omega_2$.[8]

A precise specification of the demand side structure of the labor market (the number of firms, etc.) is not important for our argument. We only assume that, at the time of hiring, employers only observe the individuals' public types, and that employers seek to maximize the expected productivity of their workforce. A labor market outcome is therefore an assignment matching individuals to jobs as a function of their gender and test score. In equilibrium of the labor market, this job-worker assignment must be ***stable*** in the following sense: No firm wants to fire one of its current workers and replace him/her with another worker currently employed in a lower ranked job. Since individuals prefer higher ranked jobs over lower ranked ones, any worker who is offered employment in a higher ranked job than his/her current one would accept the higher ranked job offer.

Given these preferences of individuals and employers, a stable assignment must be positive assortative (Becker 1973): Individuals of higher expected productivity are as-

[8]These values may include wages and monetary benefits, but also non-monetary costs and benefits such as social status or prestige associated with a job, occupational hazards, or the (un)pleasantness of working conditions. In Section 5, the assumption of fixed wages will be abandoned.

signed to higher job levels. To formalize this idea, write the expected productivity of an individual of public type $(g, \theta)$ as

$$Q_g(\theta) \;=\; \frac{\sum_{q\in K} z_g(q) f(\theta|q)\cdot q}{\widetilde{z}_g(\theta)} \;=\; \frac{\sum_{q\in K} z_g(q) f(\theta|q)\cdot q}{\sum_{q\in K} z_g(q) f(\theta|q)}. \tag{2}$$

Under (MLRP), $Q_m$ and $Q_f$ are strictly increasing in $\theta$ (Milgrom, 1981). To save on notation, we will set $Q_g(\underline{\theta}) = \lim_{\theta\to\underline{\theta}} Q_g(\theta)$ and $Q_g(\overline{\theta}) = \lim_{\theta\to\overline{\theta}} Q_g(\theta)$. Consider now cutoffs $\hat{\theta}^1_m$ and $\hat{\theta}^1_f$ such that

$$Q_m(\hat{\theta}^1_m) \begin{cases} \geq \\ = \\ \leq \end{cases} Q_f(\hat{\theta}^1_f) \;\text{ if }\; \begin{cases} \hat{\theta}^1_f & = \underline{\theta} \\ \hat{\theta}^1_m, \hat{\theta}^1_f & > \underline{\theta} \\ \hat{\theta}^1_m & = \underline{\theta} \end{cases} \quad\text{and}\quad \int_{\underline{\theta}}^{\hat{\theta}^1_m} \widetilde{z}_m(\theta)d\theta + \int_{\underline{\theta}}^{\hat{\theta}^1_f} \widetilde{z}_f(\theta)d\theta = \beta_0, \tag{3}$$

as well as cutoffs $\hat{\theta}^2_m$ and $\hat{\theta}^2_f$ such that

$$Q_m(\hat{\theta}^2_m) \begin{cases} \geq \\ = \\ \leq \end{cases} Q_f(\hat{\theta}^2_f) \;\text{ if }\; \begin{cases} \hat{\theta}^2_f & = \overline{\theta} \\ \hat{\theta}^2_m, \hat{\theta}^2_f & < \overline{\theta} \\ \hat{\theta}^2_m & = \overline{\theta} \end{cases} \quad\text{and}\quad \int_{\hat{\theta}^2_m}^{\overline{\theta}} \widetilde{z}_m(\theta)d\theta + \int_{\hat{\theta}^2_f}^{\overline{\theta}} \widetilde{z}_f(\theta)d\theta = \beta_2. \tag{4}$$

Condition (3) says that males and females with test scores below $\hat{\theta}^1_m$ and $\hat{\theta}^1_f$, respectively, comprise a measure $\beta_0$ of individuals with the lowest expected productivity conditional on their public types. In a stable assignment, these individuals must be assigned to level 0 jobs. Similarly, condition (4) says that males and females with test scores above $\hat{\theta}^2_m$ and $\hat{\theta}^2_f$ comprise a measure $\beta_2$ of individuals with the highest expected productivity. In a stable assignment, these individuals must be assigned to level 2 jobs. All individuals in between these cutoffs fill the remaining measure $\beta_1$ of level 1 jobs. Thus, a stable assignment of public types to jobs can be characterized characterized by cutoffs $\hat{\theta}^1_m, \hat{\theta}^1_f, \hat{\theta}^2_m, \hat{\theta}^2_f \in (\underline{\theta}, \overline{\theta})$ which satisfy conditions (3)–(4) above.[9]

[9]Note that conditions (3)–(4) imply a productivity threshold above which an individual is eligible for a clerical job, and a second productivity threshold above which an individual is eligible for an elite job. These thresholds (represented by the two horizontal lines in Figure 1 in Section 4) are endogenous and adjust so that the correct mass of workers $\beta_i$ is employed in each sector. This causal direction can be reversed. That is, one could consider an alternative specification of the labor market where expected productivity must be above an exogenous threshold $\overline{Q}_i$ in order to be eligible for a level-$i$ job, and where the mass of workers employed in each sector adjusts endogenously. These specifications are outcome-equivalent: For each pair of exogenous capacity constraints $(\beta_1, \beta_2)$ in the original model, there exists a pair of exogneous productivity thresholds $(\overline{Q}_1, \overline{Q}_2)$ that results in the same signal cutoffs $\hat{\theta}^1_m, \hat{\theta}^1_f, \hat{\theta}^2_m, \hat{\theta}^2_f$ (and vice versa). The bijection between $(\beta_1, \beta_2)$ and $(\overline{Q}_1, \overline{Q}_2)$ is given by (3)–(4), with $\overline{Q}_1 \equiv \min\{Q_m(\hat{\theta}^1_m), Q_f(\hat{\theta}^1_f)\}$ and $\overline{Q}_2 \equiv \min\{Q_m(\hat{\theta}^2_m), Q_f(\hat{\theta}^2_f)\}$.

### 3.4 Equilibrium

An equilibrium of this economy consists of an individual strategy $\sigma = (\sigma_m, \sigma_f)$, an assignment $\hat{\theta} = (\hat{\theta}^1_m, \hat{\theta}^1_f, \hat{\theta}^2_m, \hat{\theta}^2_f)$, and a human capital distribution $z = (z_m, z_f)$ such that three criteria are satisfied. The first is that $z$ is consistent with $\sigma$. That is, the equilibrium human capital distribution is generated by (1) from the strategy $\sigma$. The second criterion is that the labor market is in equilibrium, that is, the assignment $\hat{\theta}$ is stable. The third criterion is that the individual strategy $\sigma$ is optimal, given the assignment $\hat{\theta}$. To formalize this third criterion, denote the expected prize for an individual of type $(g, q)$ under assignment $\hat{\theta}$ by

$$W_g(q) \; = \; \omega_0 \int_{\underline{\theta}}^{\hat{\theta}^1_g} f(\theta|q)d\theta \; + \; \omega_1 \int_{\hat{\theta}^1_g}^{\hat{\theta}^2_g} f(\theta|q)d\theta \; + \; \omega_2 \int_{\hat{\theta}^3_g}^{\overline{\theta}} f(\theta|q)d\theta. \tag{5}$$

If the strategy $\sigma$ is optimal, given $\hat{\theta}$, then the following conditions hold for $g \in \{m, f\}$ and all $s \in [0, 1]$:

$$(1-\sigma_g(a))\, W_g(a) + \sigma_g(a)\big[W_g(A) - c\big] \;\; \geq \;\; (1-s)\, W_g(a) + s\big[W_g(A) - c\big], \tag{6}$$

$$(1-\sigma_g(b))\, W_g(b) + \sigma_g(b)\big[W_g(B) - c\big] \;\; \geq \;\; (1-s)\, W_g(b) + s\big[W_g(B) - c\big]. \tag{7}$$

If these conditions hold, then strategy $\sigma$ maximizes the individual's expected payoff, given the anticipated assignment $\hat{\theta}$ of workers to jobs.

An equilibrium $(\sigma, \hat{\theta}, z)$ is ***non-discriminatory*** if $\hat{\theta}^i_m = \hat{\theta}^i_f$ for $i = 1, 2$. Otherwise, it is called ***discriminatory***. In a discriminatory equilibrium, a different minimum test score is required for males, compared to females, in order to qualify for clerical and/or elite jobs. That is $\hat{\theta}^1_m \neq \hat{\theta}^1_f$ or $\hat{\theta}^2_m \neq \hat{\theta}^2_f$ or both.

**Proposition 1.** *An equilibrium exists. In particular, a non-discriminatory equilibrium exists.*

## 4 Discriminatory Equilibrium

In this section we explore the possibility of discriminatory equilibria. We focus on a particular equilibrium which features the following pure strategy for individuals:

$$\sigma_m(a) = \sigma_f(b) = 0, \;\; \sigma_m(b) = \sigma_f(a) = 1. \tag{8}$$

That is, low-ability males and high-ability females exert low effort, while high-ability males and low-ability females exert high effort. The human capital distribution in the population is then given by

$$z_m(a) = z_f(A) = \lambda, \;\; z_m(B) = z_f(b) = 1 - \lambda. \tag{9}$$

Note that, in our candidate equilibrium, the female human capital distribution is more compressed than the male human capital distribution. As we will show below, (9) implies that the equilibrium job-worker assignment $\hat{\theta}$ will be such that

$$\hat{\theta}_f^1 \ < \ \hat{\theta}_m^1 \ < \ \hat{\theta}_m^2 \ < \ \hat{\theta}_f^2. \tag{10}$$

Female workers are hence disadvantaged relative to males when it comes to obtaining an elite job, in the sense that the test score of a female worker must meet a higher threshold requirement in order to get an elite job. The opposite holds for the cutoff score needed to obtain a clerical job, where male workers are disadvantaged vis-á-vis female workers.

We will show that, under certain conditions, anticipation of the labor market assignment (10) will discourage women of high ability as well as men of low ability from investing into their human capital, yielding the strategy (8) and thus the human capital distribution (9). In this situation, more males than females will be employed in the elite sector and in simple jobs, and more females than males will be employed in clerical jobs. We call this outcome ***second-order statistical discrimination***.[10]

### 4.1 Conditions for discriminatory equilibria

Using the human capital distribution in (9), the Bayesian posterior probability that a male worker with test score $\theta$ has human capital $a$ or $B$ is

$$Pr[a|m,\theta] = \frac{\lambda f(\theta|a)}{\lambda f(\theta|a) + (1-\lambda) f(\theta|B)}, \ \ Pr[B|m,\theta] = \frac{(1-\lambda) f(\theta|a)}{\lambda f(\theta|a) + (1-\lambda) f(\theta|B)}.$$

(The human capital levels $A$ and $b$ must receive a zero probability for male workers.) The expected productivity of a male worker with test score $\theta$ is therefore given by

$$Q_m(\theta) = \frac{\lambda f(\theta|a)a + (1-\lambda) f(\theta|B)B}{\lambda f(\theta|a) + (1-\lambda) f(\theta|B)}. \tag{11}$$

Similarly, the expected productivity of this worker can be expressed as

$$Q_f(\theta) = \frac{\lambda f(\theta|A)A + (1-\lambda) f(\theta|b)b}{\lambda f(\theta|A) + (1-\lambda) f(\theta|b)}. \tag{12}$$

Because $f$ is continuous in $\theta$, the expectations $Q_m$ and $Q_f$ are continuous on $(\underline{\theta}, \overline{\theta})$. Furthermore:

**Lemma 2.** *$Q_m$ and $Q_f$ (as given in* (11)*–*(12)*) are increasing in $\theta$ and satisfy*

$$\lim_{\theta\to\underline{\theta}} Q_m(\theta) \ > \ \lim_{\theta\to\underline{\theta}} Q_f(\theta) \ > \ \lim_{\theta\to\overline{\theta}} Q_f(\theta) \ > \ \lim_{\theta\to\overline{\theta}} Q_m(\theta).$$

[10]It should be obvious that, by relabeling, one can construct another, equivalent equilibrium in which the male and female roles are reversed.

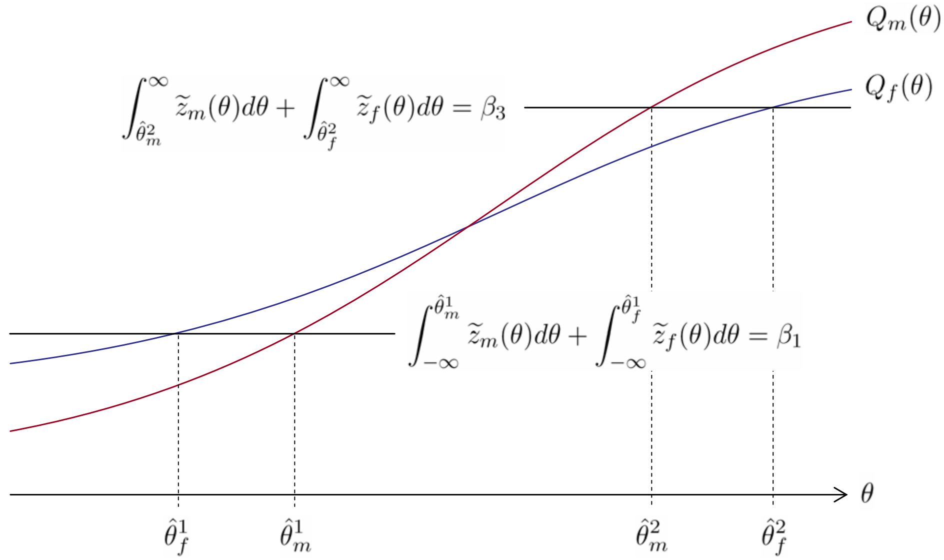


Figure 1: Assignment in discriminatory equilibrium

Because $Q_m$ and $Q_f$ are continuous in $\theta$, Lemma 2 implies that they must intersect at least once. Let $\theta^*$ be the left-most intersection of $Q_m$ and $Q_f$, and let $\theta^{**}$ be the right-most intersection. To construct the equilibrium assignment $\hat{\theta}$, we consider the following scenario:

$$\int_{\underline{\theta}}^{\theta^*} \big(\widetilde{z}_m(\theta) + \widetilde{z}_f(\theta)\big)d\theta \ > \ \beta_0, \ \ \int_{\theta^{**}}^{\overline{\theta}} \big(\widetilde{z}_m(\theta) + \widetilde{z}_f(\theta)\big)d\theta \ > \ \beta_2. \tag{13}$$

The first inequality in (13) states that the fraction of males and females with test scores below $\theta^*$ exceeds the capacity $\beta_0$. Similarly, the second inequality in (13) states that the fraction of males and females with test scores above $\theta^{**}$ exceeds the capacity $\beta_2$. Under these conditions, both males and females will be employed in the clerical sector. Some males, and possibly some females, will be employed in the elite sector as well as in the simple sector.

Cutoff thresholds $(\hat{\theta}^1_m, \hat{\theta}^1_f, \hat{\theta}^2_m, \hat{\theta}^2_m)$ to place exactly $\beta_i$ individuals into each level $i$ job can now be computed by applying conditions (3)–(4). Because $Q_m(\theta) < Q_f(\theta)$ for $\theta < \theta^*$ by Lemma 2, (3) together with the first inequality in (13) implies $\hat{\theta}^1_f < \hat{\theta}^1_m < \theta^*$. Similarly, because $Q_m(\theta) > Q_f(\theta)$ for $\theta < \theta^{**}$, (4) together with the second inequality in (13) implies $\theta^{**} < \hat{\theta}^2_m < \hat{\theta}^2_f$. The assignment hence satisfies the inequalities (10). Figure 1 illustrates the construction of this assignment graphically. The heights of the two horizontal lines are defined implicitly by (3) and (4).

Finally, we must check whether the equilibrium effort strategy posited in (8) is indeed optimal for each individual, given the assignment constructed above. Using (5)–(7), low effort is optimal for male workers with innate ability $a$ and high effort is optimal for male workers with innate ability $b$ if and only if

$$(\omega_1-\omega_0)\big[F(\hat{\theta}^1_m|a)-F(\hat{\theta}^1_m|A)\big] \;+\; (\omega_2-\omega_1)\big[F(\hat{\theta}^2_m|a)-F(\hat{\theta}^2_m|A)\big]$$
$$\leq\; c \;\leq\; (\omega_1-\omega_0)\big[F(\hat{\theta}^1_m|b)-F(\hat{\theta}^1_m|B)\big] \;+\; (\omega_2-\omega_1)\big[F(\hat{\theta}^2_m|b)-F(\hat{\theta}^2_m|B)\big]. \qquad (14)$$

The reverse inequalities hold for female workers:

$$(\omega_1-\omega_0)\big[F(\hat{\theta}^1_f|a)-F(\hat{\theta}^1_f|A)\big] \;+\; (\omega_2-\omega_1)\big[F(\hat{\theta}^2_f|a)-F(\hat{\theta}^2_f|A)\big]$$
$$\geq\; c \;\geq\; (\omega_1-\omega_0)\big[F(\hat{\theta}^1_f|b)-F(\hat{\theta}^1_f|B)\big] \;+\; (\omega_2-\omega_1)\big[F(\hat{\theta}^2_f|b)-F(\hat{\theta}^2_f|B)\big]. \qquad (15)$$

A discriminatory equilibrium therefore exists if (13) as well as (14)–(15) are satisfied, where the assignment $\hat{\theta} = (\hat{\theta}^1_m, \hat{\theta}^1_f, \hat{\theta}^2_m, \hat{\theta}^2_f)$ is derived from (3)–(4).

## 4.2 Gaussian test score distribution

In Section 4.1 we derived sufficient conditions for a discriminatory equilibrium in which the female human capital distribution had a lower variance than the corresponding male distribution. To say more, we now assume that for an individual with human capital $q \in K$, the score $\theta$ is distributed normally on $(-\infty, \infty)$ with mean $q$ and variance $\nu^2$. That is, an individual's test score is the sum of his or her human capital and Gaussian noise. The conditional density $f(\theta|q)$ is given by

$$f(\theta|q) \;=\; \frac{1}{\sqrt{2\pi}\nu} e^{-\frac{1}{2\nu^2}(\theta-q)^2}. \qquad (16)$$

Note that the Gaussian distribution satisfies both the maximum likelihood ratio property and the separation property we assumed throughout. Given the effort strategy (8), Figure 2 plots the resulting score distributions within the male and female group, assuming the fraction of individuals with low initial ability is larger than the fraction with high initial ability (i.e., $\lambda > 1/2$).

We now show, by means of an example, that equilibria with second-order statistical discrimination generically exist when test scores are normally distributed. Suppose that variance of test scores is $\nu^2 = 1/16$, and let the other parameters of our model take on the following values:

$$a = 0,\;\; A = 0.3,\;\; b = 0.6,\;\; B = 1,\;\; \beta_0 = 1,\;\; \beta_1 = 0.75,\;\; \beta_2 = 0.25,$$
$$\omega_0 = 0,\;\; \omega_1 = 1,\;\; \omega_2 = 1.5,\;\; \lambda = 0.8,\;\; c = 0.4.$$

With these parameter values, $Q_m$ and $Q_f$ cross once at 0.559. The inequalities in (13)

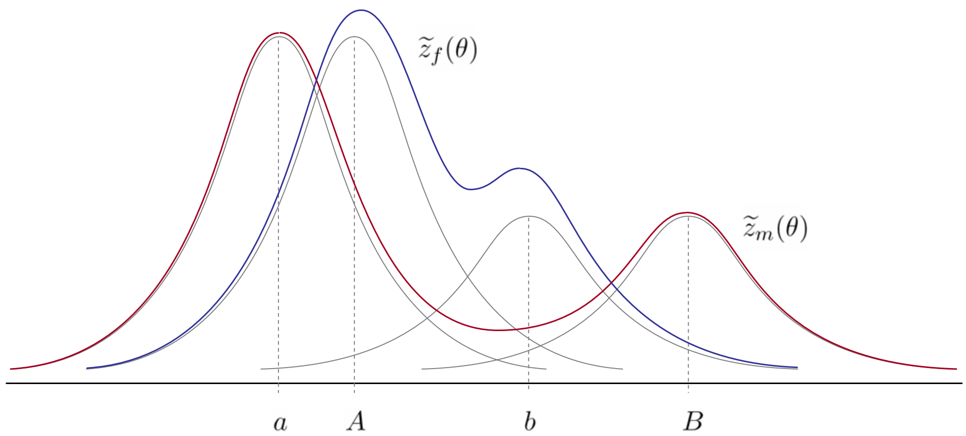


Figure 2: Gaussian test score distribution in discriminatory equilibrium

are satisfied:

$$\int_{-\infty}^{0.559} \big(\widetilde{z}_m(\theta) + \widetilde{z}_f(\theta)\big) d\theta = 1.564 \ > \ \beta_0, \quad \int_{0.559}^{\infty} \big(\widetilde{z}_m(\theta) + \widetilde{z}_f(\theta)\big) d\theta = 0.436 \ > \ \beta_2.$$

Constructing the cutoff scores for a stable assignment as in (3)–(4), we get

$$\hat{\theta}_f^1 = 0.131, \ \hat{\theta}_m^1 = 0.538, \ \hat{\theta}_m^2 = 0.582, \ \hat{\theta}_f^2 = 0.824.$$

The individual optimality condition for males is now $0.215 < c < 0.582$, and for females it is $0.459 > c > 0.317$, which implies that individuals in fact follow strategy (8). Since conditions (13) and (14)–(15) hold as strict inequalities, an equilibrium with second-order statistical discrimination exists for a non-empty open set of parameters.

In the above example, the proportion of males and females employed in each of the three job sectors is as follows:

| | Males | Females | Total |
|---|---|---|---|
| Elite | 0.199 | 0.051 | 0.25 |
| Clerical | 0.008 | 0.742 | 0.75 |
| Simple | 0.794 | 0.206 | 1 |

Furthermore, the average equilibrium human capital (and test score) for male workers this example is 0.2; while for female workers it is 0.36. It is hence possible that one group is more able on average, while another group has a higher maximum ability and will occupy a larger share of elite jobs.

### 4.3 Increasing signal precision

With perfect information about an individual's skills, of course, statistical discrimination (of any kind) cannot arise. One may therefore suspect that, if the signals about one's qualifications become more precise, the possibility of second-order statistical discrimination ceases to exist. For normally distributed test scores, however, quite the opposite is the case:

**Proposition 3.** *Assume the conditional test score distribution is normal with mean $q \in K$ and variance $\nu^2 > 0$, and suppose that the following conditions hold: $\beta_0 \in (\lambda, 2\lambda)$, $\beta_2 \in (0, 1-\lambda)$, and*

$$0 < c < \min\left\{(\omega_1 - \omega_0)\left(2 - \frac{\beta_0}{\lambda}\right), (\omega_2 - \omega_1)\frac{\beta_2}{1-\lambda}\right\}.$$

*Then there exists $\bar{\nu}^2 > 0$ such that second-order statistical discrimination is an equilibrium outcome for all $\nu^2 < \bar{\nu}^2$.*

Since access to better information improves the matching of workers to jobs by their human capital, firms seem to have an incentive to invest in the accuracy of signals they obtain about the qualifications of potential hires. As long as some arbitrarily small uncertainty about workers' qualifications remains, however, Proposition 3 implies that increasing the accuracy of signals alone does not guarantee the elimination of second-order statistical discrimination in the labor market.

One of the assumptions in Proposition 3 is that the number of elite jobs is less than the number of high-ability males. Since females have at most human capital $b < B$ in the examined human capital distribution, and males with human capital $B$ can signal this fact to an arbitrarily large degree of precision as $\nu^2$ becomes small, the elite sector will be filled exclusively with males in the equilibria in Proposition 3. Thus, female workers will be even more severely discriminated against when test scores become increasingly precise signals.[11]

# 5 Endogenous Wages

Note that an agent has, in principle, two reasons to invest in human capital. The first is to increase his or her chance of being assigned to a higher job level. The second is to increase his or her wage within a job level. In the basic model presented in Section 3

[11] On the other hand, it is never possible to have an impenetrable "glass bottom" at the same time, with no female workers being employed in the simple sector. If this was the case, then all females would be employed in the clerical sector, regardless of their test score, and no incentive would exist for any female to invest in human capital production. The male human capital distribution would thus at least weakly dominate the female human capital distribution, which means that at least as many females as males must be employed in the simple sector—a contradiction.

only the first incentive mattered, as an individual's wage was a step function of his or her test score:

$$\text{Wage}(\theta, g) \;=\; \begin{cases} \omega_0 & \text{if } \theta < \hat{\theta}_g^1, \\ \omega_1 & \text{if } \hat{\theta}_g^1 \le \theta < \hat{\theta}_g^2, \\ \omega_2 & \text{if } \hat{\theta}_g^2 \le \theta. \end{cases}$$

This compensation structure meant that individuals were competing *for* jobs, but not *within* job sectors.

We now present an extension of our model where wage income does depend on a worker's expected human capital. To this end, let us assume that an individual of human capital $q$ produces value $\rho_i q$ when assigned to a job of level $i \in \{0, 1, 2\}$, with $\rho_2 > \rho_1 > \rho_0$. In addition, we must specify a wage setting process through which this value is split between employers and employees. In order to generate the same investment incentives as before, we require a model of wage setting that can generate sufficiently large differences in compensation *across* job levels, relative to any variation in compensation *within* job levels. That is, the first type of incentive discussed in the beginning must dominate the second. The following wage setting process demonstrates one possibility of how such a pay schedule may arise.

Suppose that production takes place in a single firm, which has $\beta_i$ openings at job level $i$ ($i = 0, 1, 2$). After the individuals have made their effort investments, the firm observes each individual's test score $\theta$ and assigns him or her to a job. This assignment is fixed and cannot be negotiated by the worker; however, the firm negotiates with each individual the compensation he or she will receive. If public type $(g, \theta)$ is assigned to job level $i$, the expected value generated by this match is $\rho_i \cdot Q_g(\theta)$. Assume any bargaining solution which lets the individual keep a fraction $\alpha$ of this value, and the firm a fraction $1 - \alpha$.[12] With this specification, the firm will match applicants to jobs by their expected productivities, as was the case in the basic model. Assuming the same human capital distribution in (9), the job worker assignment is still characterized by cutoffs $\hat{\theta}_f^1 < \hat{\theta}_m^1 < \hat{\theta}_m^2 < \hat{\theta}_f^2$ which satisfy conditions (3)–(4).[13]

[12] This would be the outcome predicted by the Nash bargaining solution with bargaining powers $\alpha$ and $1 - \alpha$, respectively. Alternatively, it would be the approximate split of the value in equilibrium of the Rubinstein-Stahl bargaining game with discount factors $\delta_W$ for workers and $\delta_F$ for firms, with $\delta_W \approx 1 \approx \delta_F$ and $(1-\delta_W)/(1-\delta_F) = (1-\alpha)/\alpha$.

[13] Note that this bargaining solution generates discrete jumps in the wages paid to individuals whose signals are just above these cutoffs. These jumps are important, since they ensure that the "competition for jobs" investment incentive dominates the "competition within jobs" incentive. Bertrand wage competition, on the other hand, would result in continuous wages. (To see this, consider a man whose signal is just below $\hat{\theta}_m^2$. If there was a jump in male wages at this cutoff, this worker would be able to undercut the wage of male workers with signals just above $\hat{\theta}_m^2$. Since these workers are only slightly more productive than the worker with signal just below $\hat{\theta}_m^2$, firms would prefer to hire the marginally less productive worker.) As discussed above, our discriminatory equilibrium relies on the "competition for jobs" incentive, and with continuous wages this incentive would go away.

The expected payoff for an individual of private type $(g,q)$ now becomes

$$W_g(q) = \alpha\left[\rho_0\int_{\underline{\theta}}^{\hat{\theta}_g^1} Q_g(\theta)f(\theta|q)d\theta + \rho_1\int_{\hat{\theta}_g^1}^{\hat{\theta}_g^2} Q_g(\theta)f(\theta|q)d\theta + \rho_2\int_{\hat{\theta}_g^2}^{\overline{\theta}} Q_g(\theta)f(\theta|q)d\theta\right]. \quad (17)$$

We thus have an equilibrium with second-order statistical discrimination if $W_m(A) - W_m(a) < c < W_m(B) - W_m(b)$ and $W_m(A) - W_m(a) > c > W_m(B) - W_m(b)$. The following example shows that, generically, this is possible.

Consider again the Gaussian test score distribution (16) with variance $\nu^2 = 1/16$, and suppose the parameters of the (extended) model take on the following values:

$$a = 0,\ \ A = 0.3,\ \ b = 0.6,\ \ B = 1,\ \ \beta_0 = 1,\ \ \beta_1 = 0.9,\ \ \beta_2 = 0.1,$$
$$\rho_0 = 0.5,\ \ \rho_1 = 2,\ \ \rho_2 = 4,\ \ \lambda = 0.8,\ \ c = 0.12.$$

As in the example discussed in Section 4.2, $Q_m$ and $Q_f$ cross at 0.559. Condition (13) continues to hold:

$$\int_{-\infty}^{0.559}\big(\widetilde{z}_m(\theta) + \widetilde{z}_f(\theta)\big)d\theta = 1.546 > \beta_0, \quad \int_{0.559}^{\infty}\big(\widetilde{z}_m(\theta) + \widetilde{z}_f(\theta)\big)d\theta = 0.436 > \beta_2.$$

The stable assignment is now characterized by new cutoff scores,

$$\hat{\theta}_f^1 = 0.131,\ \hat{\theta}_m^1 = 0.538,\ \hat{\theta}_m^2 = 1.000,\ \hat{\theta}_f^2 = \infty.$$

That is, it is impossible for females to obtain elite jobs. The individual optimality condition for males becomes $0.117 < c < 0.881$, and for females it becomes $0.135 > c > 0.113$. Thus, individuals in fact follow strategy (8). The proportion of males and females in each of the three job sectors is as follows:

| | Males | Females | Total |
|---|---|---|---|
| Elite | 0.100 | 0.000 | 0.1 |
| Clerical | 0.106 | 0.794 | 0.9 |
| Simple | 0.794 | 0.206 | 1.0 |

Furthermore, the average wage for male workers in this equilibrium is $\lambda W_m(a) + (1-\lambda)W_m(B) = 0.295$, and the average wage for female workers is $\lambda W_f(A) + (1-\lambda)W_f(b) = 0.442$. Thus, like first-order statistical discrimination, second-order discrimination can lead to differences in mean wages across groups. Unlike first-order discrimination, however, second-order discrimination does not have a uniform impact on wages of all members within a group.

## 6 Conclusion

This paper extended the self-fulfilling expectations model of statistical discrimination from the first to the second moment of the skill distribution. The differential treatment of groups by employers—which we called second-order statistical discrimination—manifested itself in the disproportionate representation of groups at different job levels Fewer women than men were employed in the simple and the elite sector of the job market, and more women than men were employed in the middle (clerical) sector.

Like every model of statistical discrimination, ours relies on an informational friction which prevents employers from learning an individual's skills perfectly. This friction induces a two-fold inefficiency: First, it discourages some high-ability individuals from investing, while some low-ability individuals invest. Because the cost of investing is assumed to be constant but the returns are larger for high-ability individuals, this is a misallocation of resources. Second, taken the investment decisions as given, the final allocation of workers to jobs may not be optimal. It is questionable whether markets can correct these efficiencies by themselves. As shown in Proposition 3, even when signals become increasingly precise, second-order statistical discrimination remains an equilibrium. Thus, some policy interventions are needed to correct the inefficiencies resulting from second-order statistical discrimination.

It is beyond the scope of this work to examine the many conceivable policy measures here. However, our results allow us to at least speculate on some aspects of policy. First, the presence of second-order statistical discrimination may remain undetected when comparing average outcomes across groups. For instance, a comparison of average wages across men and women may suggest equality of *average* male and female labor market outcomes. However, second-order statistical discrimination tends to have a different impact on different individuals in the same group, and as is often the case, the first moment of a distribution does not convey all relevant information contained in this distribution. In the context of our model, this means that comparing group-wide average labor market outcomes does not allow detection of more subtle forms of discrimination.

Second, while we cannot conclude from our simple theoretical model precisely which corrective measures work best in practice, the differential impact of second-order statistical discrimination on individuals within the same group suggests that policies should be targeted at certain jobs, individuals, or firms only. For example, a broad-stroke mandate that firms employ an equal number of men and women would have no effect on firms which offer all three tiers of jobs. On the other hand, a mandate that an equal number of men and women be employed in elite jobs would have an equalizing effect on male and female employment in this sector (and might then result in altered investment incentives for high-ability females).

Third, policies targeting the elite job sector may have spill-over effects to lower job sectors. An equal-employment mandate on the elite job level will reallocate some women from the clerical sector to the top sector, and some men from the top sector the clerical sector. These changes in the clerical sector could, in turn, affect the assignment of workers to the simple sector. Thus, a highly targeted policy intervention can potentially affect the labor market outcomes of all individuals, and hence the investment incentives of all individuals.

# Appendix

## Proof of Proposition 1

Without loss of generality we can take the support of the test score distribution $f$ to be a subset of $[0,1]$, if necessary by constructing a new test score $t \in [0,1]$ and setting $t \equiv e^{\theta}/(1+e^{\theta})$. An equilibrium is then a point $(\sigma, \hat{\theta}, z) \in S \equiv [0,1]^4 \times [0,1]^4 \times \Delta^2$, satisfying the conditions in Section 3.4, where

$$\Delta \equiv \Big\{ \big(z(a), z(A), z(b), z(B)\big) \in [0,1]^4 \;:\; z(a)+z(A)+z(b)+z(B)=1 \Big\}$$

is the set of human capital distributions within each group. Define three correspondences

$$T^{\sigma} : [0,1]^4 \rightarrow\rightarrow [0,1]^4, \quad T^{\hat{\theta}} : \Delta^2 \rightarrow\rightarrow [0,1]^4, \quad T^{z} : [0,1]^4 \rightarrow\rightarrow \Delta^2$$

as follows:

$$\begin{array}{rcllll} T^{\sigma}(\hat{\theta}) & = & \{ & \sigma' \in [0,1]^4 & : & s' \text{ is optimal given } \hat{\theta} \quad \}, \\ T^{\hat{\theta}}(z) & = & \{ & \hat{\theta}' \in [0,1]^4 & : & \hat{\theta}' \text{ is stable given } z \quad \}, \\ T^{z}(\sigma) & = & \{ & z' \in \Delta^2_K & : & z' \text{ is consistent with } \sigma \quad \}. \end{array}$$

The definitions of optimality, stability, and consistence (given in Section 3.3 and 3.4) imply that $T^{\sigma}$, $T^{\hat{\theta}}$, $T^{z}$ are all upper-hemicontinuous. Define a new correspondence $T : S \rightarrow\rightarrow S$ by setting $T(\sigma, \hat{\theta}, z) \equiv T^{\sigma}(\hat{\theta}) \times T^{\hat{\theta}}(z) \times T^{z}(\sigma)$. Then $T$ is upper-hemicontinuous on a compact and convex set $S \subset \mathbb{R}^{16}$. By Kakutani's Fixed Point Theorem there exists a point $(\sigma, \hat{\theta}, z) \in T(\sigma, \hat{\theta}, z)$, satisfying our equilibrium conditions.

To show that a non-discriminatory equilibrium exists, let $\hat{S} \subset S$ be defined as

$$\hat{S} \equiv \{ \; (\sigma, \hat{\theta}, z) \in S \; : \; \sigma_m = \sigma_f, \, \hat{\theta}^1_m = \hat{\theta}^1_f, \, \hat{\theta}^2_m = \hat{\theta}^2_f, \, z_m = z_f \; \}.$$

Observe that $\hat{S}$ is a compact and convex subset of $\mathbb{R}^{16}$. Furthermore, $T(\sigma, \hat{\theta}, z) \cap \hat{S} \neq \emptyset$ for all $(\sigma, \hat{\theta}, z) \in \hat{S}$. To see this, make the following three observations:

1. If $\hat{\theta}^1_m = \hat{\theta}^1_f$ and $\hat{\theta}^2_m = \hat{\theta}^2_f$, an effort decision is optimal for males if and only if the same decision is optimal for females; thus there exists an optimal strategy $s = (s_m, s_f)$ with $s_m = s_f$.

2. If $s_m = s_f$, the human capital distribution among males is the same as among females: $z_m = z_f$.

3. If $z_m = z_f$, then $Q_m = Q_f$ and a stable assignment in the labor market will be such that $\hat{\theta}^1_m = \hat{\theta}^1_f$ and $\hat{\theta}^2_m = \hat{\theta}^2_f$.

Thus, $T$ can be restricted to an upper-hemicontinuous correspondence from $\hat{S}$ into $\hat{S}$. By Kakutani's Fixed Point Theorem, a fixed point in $\hat{S}$ exists, which then satisfies the definition of a non-discriminatory equilibrium. □

## Proof of Lemma 2

Define

$$\gamma_m(\theta) \equiv \left[1 + \frac{1-\lambda}{\lambda}\frac{f(\theta|B)}{f(\theta|a)}\right]^{-1}, \quad \gamma_f(\theta) \equiv \left[1 + \frac{1-\lambda}{\lambda}\frac{f(\theta|b)}{f(\theta|A)}\right]^{-1}.$$

We first claim that, under (MLRP) $\gamma_m(\theta)$ and $\gamma_f(\theta)$ satisfy the following properties:

(i) $\gamma_m(\theta)$ and $\gamma_f(\theta)$ are weakly decreasing in $\theta$,

(ii) $\lim_{\theta\to\underline{\theta}} \gamma_m(\theta) > \lim_{\theta\to\overline{\theta}} \gamma_m(\theta)$ and $\lim_{\theta\to\underline{\theta}} \gamma_f(\theta) > \lim_{\theta\to\overline{\theta}} \gamma_f(\theta)$,

(ii) $\lim_{\theta\to\underline{\theta}} \gamma_m(\theta) > \lim_{\theta\to\underline{\theta}} \gamma_f(\theta)$ and $\lim_{\theta\to\overline{\theta}} \gamma_m(\theta) > \lim_{\theta\to\overline{\theta}} \gamma_f(\theta)$.

To show (i), note that (MLRP) implies $f(\theta|B)/f(\theta|a)$ and $f(\theta|b)/f(\theta|A)$ are increasing in $\theta$. Thus, $\gamma_m(\theta)$ and $\gamma_f(\theta)$ are decreasing in $\theta$. To show (ii), suppose $\lim_{\theta\to\underline{\theta}} \gamma_m(\theta) \le \lim_{\theta\to\overline{\theta}} \gamma_m(\theta)$. By claim (i), this implies $\gamma_m(\theta)$ is a constant for all $\theta$, which in turn implies that $f(\theta|B)/f(\theta|a)$ is independent of $\theta$. This is not possible, due to the stricty inequality part of (MLRP). To show (iii), express $f(\theta|B)/f(\theta|a)$ as

$$\frac{f(\theta|B)}{f(\theta|a)} = \frac{f(\theta|b)}{f(\theta|A)} \cdot \frac{f(\theta|A)}{f(\theta|a)}\frac{f(\theta|B)}{f(\theta|b)}$$

and observe that, as $\theta$ decreases, $f(\theta|A)/f(\theta|a)$ and $f(\theta|B)/f(\theta|b)$ are decreasing due to (MLRP). Furthermore, both terms must fall below one for $\theta$ small enough. To see why, suppose to the contrary that $f(\theta|A)/f(\theta|a) \ge 1$ as $\theta \to \underline{\theta}$. This implies $f(\theta|A) \ge f(\theta|a)$ for all $\theta$ due to (MLRP). Furthermore, $f(\theta|A) < f(\theta|a)$ on an open set of values for $\theta$, due to the strict inequality part of (MLRP) and continuity of $f$. But then $1 = \int f(\theta|A)d\theta > \int f(\theta|a)d\theta = 1$, a contradiction. The same argument applies to $f(\theta|B)/f(\theta|b)$. It follows that $f(\theta|B)/f(\theta|a) < f(\theta|b)/f(\theta|A)$ as $\theta \to \underline{\theta}$, and therefore $\lim_{\theta\to\underline{\theta}} \gamma_m(\theta) > \lim_{\theta\to\underline{\theta}} \gamma_f(\theta)$. The second inequality can be shown in the same fashion.

Now express $Q_m$ and $Q_f$ as follows:

$$Q_m(\theta) = \gamma_m(\theta)a + [1-\gamma_m(\theta)]\,B, \tag{18}$$

$$Q_f(\theta) = \gamma_f(\theta)A + [1-\gamma_f(\theta)]\,b. \tag{19}$$

Since $a < B$ and $A < b$, claim (i) implies $Q_m$ and $Q_f$ are increasing in $\theta$. Next, we show that

$$\lim_{\theta\to\underline{\theta}} Q_m(\theta) < \lim_{\theta\to\underline{\theta}} Q_f(\theta) < \lim_{\theta\to\overline{\theta}} Q_f(\theta) < \lim_{\theta\to\overline{\theta}} Q_m(\theta).$$

The middle inequality follows from claim (ii). To show the left inequality, note that (SEP) implies

$$1+\frac{1-\lambda}{\lambda}\lim_{\theta\to\underline{\theta}}\frac{f(\theta|B)}{f(\theta|a)} \;<\; 1+\frac{A-a}{B-b}$$

and thus $\lim_{\theta\to\underline{\theta}}\gamma_m(\theta)\cdot(B-b+A-a)>B-b$. By claim (iii) and $b>A$ it follows that $\lim_{\theta\to\underline{\theta}}\gamma_m(\theta)\cdot(B-a)-\lim_{\theta\to\underline{\theta}}\gamma_f(\theta)\cdot(b-A)>B-b$. Rearranging, we get

$$\lim_{\theta\to\underline{\theta}}\gamma_m(\theta)\cdot a+\left[1-\lim_{\theta\to\underline{\theta}}\gamma_m(\theta)\right]\cdot B \;<\; \lim_{\theta\to\underline{\theta}}\gamma_f(\theta)\cdot A+\left[1-\lim_{\theta\to\underline{\theta}}\gamma_f(\theta)\right]\cdot b$$

or $\lim_{\theta\to\underline{\theta}}Q_m(\theta)<\lim_{\theta\to\underline{\theta}}Q_f(\theta)$, as desired. The inequality $\lim_{\theta\to\bar{\theta}}Q_f(\theta)<\lim_{\theta\to\bar{\theta}}Q_m(\theta)$ can be shown in a similar manner. □

## Proof of Proposition 3

Express $Q_m(\theta)$ and $Q_f(\theta)$ as in (18)–(19). Using the normal distribution for $\theta$, the weights $\gamma_m(t)$ and $\gamma_f(\theta)$ can be written as

$$\begin{aligned}\gamma_m(\theta) &= \left[1+\frac{1-\lambda}{\lambda}\exp\Big(-\frac{1}{2\nu^2}(B-a)(B+a-2\theta)\Big)\right]^{-1},\\ \gamma_f(\theta) &= \left[1+\frac{1-\lambda}{\lambda}\exp\Big(-\frac{1}{2\nu^2}(b-A)(b+A-2\theta)\Big)\right]^{-1}.\end{aligned}$$

Note that that $\gamma_m((a+B)/2)=\gamma_f((A+b)/2)=\lambda$. Observe further that for every $\delta>0$ one can find $\varepsilon>0$ such that $\nu^2<\varepsilon$ implies $\gamma_m(\theta)>1-\delta$ for all $\theta<(B+a)/2-\delta$, as well as $\gamma_m(\theta)<\delta$ for all $\theta>(B+a)/2+\delta$. Similarly, for every $\delta>0$ one can find $\varepsilon>0$ such that $\nu^2<\varepsilon$ implies $\gamma_f(\theta)>1-\delta$ for all $\theta<(b+A)/2-\delta$, as well as $\gamma_f(\theta)<\delta$ for all $\theta>(b+A)/2+\delta$. As $\nu^2\to 0$, therefore, the expectations $Q_m$ and $Q_f$ converge pointwise to

$$\lim_{\nu^2\to 0}Q_m(\theta) \;=\; \begin{cases} a & \text{if } \theta<(a+B)/2,\\ \lambda a+(1-\lambda)B & \text{if } \theta=(a+B)/2,\\ B & \text{if } \theta>(a+B)/2\end{cases} \tag{20}$$

and

$$\lim_{\nu^2\to 0}Q_f(\theta) \;=\; \begin{cases} A & \text{if } \theta<(A+b)/2,\\ \lambda A+(1-\lambda)b & \text{if } \theta=(A+b)/2,\\ b & \text{if } \theta>(A+b)/2.\end{cases} \tag{21}$$

(For small but positive $\nu^2$, the expectations $Q_m$ and $Q_f$ are depicted in Figure 3, and as $\nu^2$ decreases further, these curves will increasingly look like the limiting functions given in (20)–(21).) Note that, for $\nu^2\approx 0$, $Q_m$ and $Q_f$ intersect once at $\theta^*\approx(a+B)/2$. Note also that $B-b>A-a$ implies $(a+B)/2>(A+b)/2$, and that $b>A$ implies $(A+b)/2>A$.

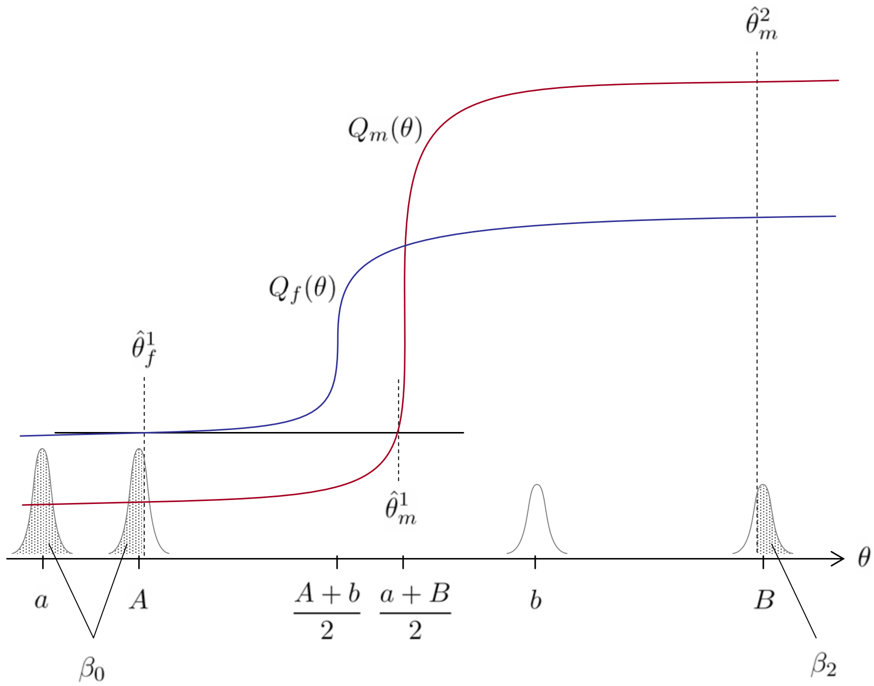


Figure 3: Construction of a discriminatory equilibrium with small Gaussian noise

Next, we construct the job-worker assignment $\hat{\theta}$ and examine its properties for small $\nu^2$. (Figure 3 also illustrates the construction of this assignment.) Throughout, if $x$ and $y$ are two variables, the statement "$x \approx y$ for $\nu^2 \approx 0$" means "$\forall \delta > 0\ \exists \varepsilon > 0$ s.t. $\nu^2 < \varepsilon \Rightarrow |x - y| < \delta$." Note that the distribution of test scores, $(\widetilde{z}_m, \widetilde{z}_f)$, is concentrated around the human capital levels $a, A, b, B$, and increasingly so as $\nu^2 \to 0$. For $\nu^2 \approx 0$, therefore, we have

$$\int_{-\infty}^{\theta^*} \big(\widetilde{z}_m(\theta) + \widetilde{z}_f(\theta)\big) d\theta \ \approx\ 2\lambda, \quad \int_{\theta^*}^{\infty} \big(\widetilde{z}_m(\theta) + \widetilde{z}_f(\theta)\big) d\theta \ \approx\ 2(1-\lambda).$$

Given $\beta_0 \in (\lambda, 2\lambda)$ and $\beta_2 \in (0, 1-\lambda)$, condition (13) for a second-order discriminatory equilibrium holds for sufficiently small $\nu^2$. Furthermore, condition $\beta_0 \in (\lambda, 2\lambda)$ means that there are more simple jobs than males with human capital $a$, but fewer than males with human capital $a$ and females with human capital $A$ combined. For $\nu^2 \approx 0$, this implies that approximately $\lambda$ males and approximately $\beta_0 - \lambda < \lambda$ of females will be employed in the simple sector. The cutoff scores that separate simple jobs from clerical jobs therefore satisfy $\hat{\theta}^1_f \approx A < (A+b)/2 < (a+B)/2 \approx \hat{\theta}^1_m$. On the other hand, condition $\beta_2 \in (0, 1-\lambda)$ means that there are fewer elite jobs than males with human capital $B$. For $\nu^2 \approx 0$, this implies that exactly $\beta_2$ males and no females will be employed in the elite sector. The cutoff scores that separate clerical jobs from elite jobs therefore satisfy $(B+a)/2 < B \approx \hat{\theta}^2_m < \hat{\theta}^2_f = \infty$. For small enough $\nu^2$, the job-worker assignment therefore satisfies $\hat{\theta}_f^1 < \hat{\theta}_m^1 < \hat{\theta}_m^2 < \hat{\theta}_f^2$, as required in a second-order discriminatory equilibrium.

Finally, we examine the individual incentives to invest effort, under the assignment constructed above and again assuming that $\nu^2 \approx 0$. First, consider males of initial ability $a$. For $\nu^2 \approx 0$, we have $F(\hat{\theta}_m^1|a) \approx F(\hat{\theta}_m^1|A) \approx F(\hat{\theta}_m^2|a) \approx F(\hat{\theta}_m^2|A) \approx 1$. The expression on the left side of (14) therefore becomes

$$(\omega_1-\omega_0)\big[F(\hat{\theta}_m^1|a)-F(\hat{\theta}_m^1|A)\big] + (\omega_2-\omega_1)\big[F(\hat{\theta}_m^2|a)-F(\hat{\theta}_m^2|A)\big] \;\approx\; 0.$$

Second, consider males of initial ability $b$. For $\nu^2 \approx 0$, we have $F(\hat{\theta}_m^1|b) \approx F(\hat{\theta}_m^1|B) \approx 0$, $F(\hat{\theta}_m^2|b) \approx 1$, and $F(\hat{\theta}_m^2|B) \approx 1 - \beta_2/(1-\lambda)$. The expression on the right side of (14) therefore becomes

$$(\omega_1-\omega_0)\big[F(\hat{\theta}_m^1|b)-F(\hat{\theta}_m^1|B)\big] + (\omega_2-\omega_1)\big[F(\hat{\theta}_m^2|b)-F(\hat{\theta}_m^2|B)\big] \;\approx\; (\omega_2-\omega_1)\frac{\beta_2}{1-\lambda}.$$

Third, consider females of initial ability $a$. For $\nu^2 \approx 0$, we have $F(\hat{\theta}_f^1|a) \approx 1$, $F(\hat{\theta}_f^1|A) \approx \beta_0\lambda - 1$, and $F(\hat{\theta}_f^2|a) \approx F(\hat{\theta}_f^2|A) \approx 1$. The expression on the left side of (15) therefore becomes

$$(\omega_1-\omega_0)\big[F(\hat{\theta}_f^1|a)-F(\hat{\theta}_f^1|A)\big] + (\omega_2-\omega_1)\big[F(\hat{\theta}_f^2|a)-F(\hat{\theta}_f^2|A)\big] \;\approx\; (\omega_1-\omega_0)\Big(2-\frac{\beta_0}{\lambda}\Big).$$

Finally, consider females of initial ability $b$. For $\nu^2 \approx 0$, we have $F(\hat{\theta}_f^1|b) \approx F(\hat{\theta}_f^1|B \approx 0$ and $F(\hat{\theta}_f^2|b) \approx F(\hat{\theta}_f^2|B) \approx 1$. The expression on the right side of (15) therefore becomes

$$(\omega_1-\omega_0)\big[F(\hat{\theta}_f^1|b)-F(\hat{\theta}_f^1|B)\big] + (\omega_2-\omega_1)\big[F(\hat{\theta}_f^2|b)-F(\hat{\theta}_f^2|B)\big] \;\approx\; 0.$$

If follows that, if $0 < c < \min\{(\omega_1-\omega_0)(2-\beta_0/\lambda), (\omega_2-\omega_1)\beta_2/(1-\lambda)\}$, all inequalities in (14)–(15) are satisfied for sufficiently small $\nu^2 > 0$. □

## References


[1] Aigner, Dennis and Glen Cain. 1977. "Statistical Theories of Discrimination in Labor Markets." *Industrial and Labor Relations Review*, **30**, 175–187.

[2] Arrow, Kenneth. 1973. "The Theory of Discrimination." In: Orley Ashenfelter and Albert Rees (eds.): *Discrimination in Labor Markets*. Princeton, NJ: Princeton University Press. 3–33.

[3] Becker, Gary. 1957. *The Economics of Discrimination*. Chicago, IL: University of Chicago Press.

[4] Becker, Gary. 1973. "A Theory of Marriage: Part I." *Journal of Political Economy*, **81**, 813–846.

[5] Becker, Gary, Kevin Murphy, and Iván Werning. 2005. "The Equilibrium Distribution of Income and the Market for Status." *Journal of Political Economy*, **113**, 282–301.

[6] Bjerk, David. 2007. "The Differing Nature of Black-White Wage Inequality Across Occupational Sectors." *Journal of Human Resources*, **42**, 398–434.

[7] Bjerk, David. 2008. "Glass Ceilings or Sticky Floors? Statistical Discrimination in a Dynamic Model of Promotion and Hiring." *The Economic Journal*, **118**, 961–982.

[8] Browne, Kingsley. 1998. "An Evolutionary Account of Women's Workplace Status." *Managerial and Decision Economics*, **19**, 427–440.

[9] Browne, Kingsley. 2006. "Evolved Sex Differences and Occupational Segregation." *Journal of Organizational Behavior*, **27**, 143–162.

[10] Coate, Stephen and Glenn Loury. 1993. "Will Affirmative-Action Policies Eliminate Negative Stereotypes?" *American Economic Review*, **83**, 5, 1220–1240.

[11] Fang, Hanming and Andrea Moro. 2011. "Theories of Statistical Discrimination and Affirmative Action: A Survey." In Jess Benhabib, Matthew O. Jackson, and Alberto Bisin, editors: *Handbook of Social Economics, Vol. 1A.* The Netherlands: North Holland.

[12] Lundberg, Shelly and Richard Startz. 1983. "Private Discrimination and Social Intervention in Competitive Labor Markets." *American Economic Review*, **73**, 340–347.

[13] Milgrom, Paul. 1981. "Good News and Bad News: Representation Theorems and Applications." *Bell Journal of Economics*, **12**, 380–391.

[14] Moro, Andrea. 2003. "The Effect of Statistical Discrimination on Black-White Wage Inequality: Estimating a Model with Multiple Equilibria." *International Economic Review*, **44**, 467–500.

[15] Phelps, Edmund. 1972. "The Statistical Theory of Racism and Sexism." *American Economic Review*, **62**, 4, 659–661.

[16] Ray, Debraj and Arthur Robson. 2011. "Status, Intertemporal Choice and Risk-Taking." mimeo.

[17] Rubin, Paul and Chris Paul. 1979. "An Evolutionary Model of Taste for Risk." *Economic Inquiry*, **17**, 585–696.

[18] Summers, Lawrence. 2005. Remarks at NBER Conference on Diversifying the Science & Engineering Workforce, January 14, 2005. `http://www.president.harvard.edu/speeches/2005/nber.html`