# Modeling and identifying central banks' preferences.*

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**Abstract**

We propose an approach to identify independently the parameters describing the structure of the economy from those describing central bank preferences. We first estimate a parsimonious structural model for US inflation, US output-gap and the world commodity price index. We then proceed to the identification of central bank preferences by estimating by GMM the Euler equations for the solution of the intertemporal optimization problem relevant to the central bank. The empirical analysis of the structural model shows that the persistency of real interest rates effects on aggregate demand is sufficient to generate an autoregressive structure in any interest rate rule. From estimation of the Euler equations, we infer that strict inflation targeting together with real interest rate smoothing delivers an optimal policy rule capable of replicating the observed path of real interest rates over the sample 1983:1-1998:3. Our empirical findings imply that the output gap enters into the optimal interest rate rule only as a leading indicator of future inflation, and we reject the hypothesis that output stabilization is an independent argument in the loss function of the Fed.

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# 1. Introduction

A growing body of empirical literature has documented the usefulness of interest rate rules as a convenient way to model and interpret central banks' policy. Interest rate rules relate the setting of a short-term money market rate (such as the Federal Funds rate in the US) to a central bank's perception of the inflation and output gaps. The plausibility of such rules has either been simply postulated (as in Taylor, 1993, from which this literature originates), or may be derived by solving the intertemporal optimization problem for a suitable loss function. In the latter case, derivation of the interest rate rule makes it clear that estimated coefficients can only be interpreted as convolutions of the parameters describing central bank preferences and of those determining the structure of the economy. This is a clear instance of the well known Lucas' critique (Lucas,1976). But, although this point is clear in theory (see e.g. Svensson,1996), somewhat surprisingly there are very few reported attempts in the literature (Lippi,1998 Ch.8, Cecchetti et al.,1998) to recover the preferences parameters of the central bank from estimated interest rate rules. This is even more surprising if one considers that a large and influential strand of literature (originating with Barro and Gordon, 1986) has been built on the untested assumption that central bank preference for output stabilization may be at the origin of observed inflation in many countries. Recently, some authors (Posen, 1993; Blinder,1997) have expressed a different opinion, suggesting that time inconsistency might not, after all, be the most accurate characterization of central banks' decision making. Thus we believe that it would be of some interest, to both sides of this controversy, if some more direct evidence on the determinants of central banks policy choices were available.

On the empirical side a consensus view has emerged that estimated forward looking interest rate rules are generally consistent with the "inflation targeting" approach, and the theoretical models put forward by Svensson and coauthors in a number of papers (Svensson, 1997, 1998, Rudebusch and Svensson,1998) give the theoretical foundations for the empirical estimation. One point which is left unresolved in this literature is, having observed that central banks do react to observed output gaps (see e.g, for the case of Germany, Clarida, Gali and Gertler, 1997), whether the finding that central banks do set interest rates in response to both expected inflation and expected output gaps is *compatible* with the (often stated as prioritary) inflation-control objective, or whether it is indeed *instrumental* to it (as it would be the case in the event of aggregate demand shocks (See e.g. Svensson, 1996, and Goodfriend and King, 1997), or both. To settle this issue, again, availability of more direct evidence on whether central banks decisions are the outcome of trading off output vs. inflation stabilization would be useful.

A related problem is that, to be able to match the data, simulated or estimated interest rate rules have always needed to include a term in the lagged interest rate. This can be rationalized by assuming some partial adjustment mechanism between current interest rates and the equilibrium rate or - which is perhaps less arbitrary - by assuming that interest rate smoothing enters as an additional explicit objective into central banks' preferences (as argued in Goodfriend, 1987). However, this point is really open to debate. For instance Sack (1998) has argued that the persistence of policy rates can instead be related to persistence in the structure of the economy while Brainard type uncertainty could explain the observed smooth response of policy rates to macroeconomic conditions. Also on this issue we think that availability of direct evidence might be helpful to settle the debate.

Accordingly, the first aim of this paper is to propose an approach which allows to identify independently the parameters describing central bank preferences from those characterizing the structure of the economy. This way we can explore to what extent the policy decisions of a central bank have been motivated, if at all, by the desire to stabilize output as an objective in its own right[1]. Having shown the potential of our methodology, we precisely formulate and test alternative hypotheses justifying the observed persistence or sluggishness in the setting of interest rates.

Our model is applied to the US, as this has been both the most widely studied case so far and it is the most natural (and perhaps only) example of an (almost) closed economy. We first estimate the parameters describing the structure of the economy by considering a parsimonious specification for inflation, the output-gap and the commodity price index. Our choice of variables is driven by the wide consensus on the minimal set of macroeconomic variables to analyze monetary policy making (see, for example, Christiano et al.,1998). We then proceed to the identification of central bank preferences by GMM estimation of the Euler equations obtained as a solution to the intertemporal optimization problem relevant to the central bank. While previous literature has used the GMM methodology to estimate central banks' reaction functions, we observe that the natural object

[1] Although we do not discuss whether the discretionary behavior of the central bank leads to the emergence of an inflation bias (in fact we assume away this issue by setting potential output as the target level for output), in principle our methodology could be extended to address this issue empirically.

of GMM estimation are instead the first order conditions derived from the central banks' optimization problem. Finally, we compare optimal and actual interest rate behavior, to select a structure of central bank's preferences capable of delivering the observed behavior of policy rates.

Our paper is structured as follows. In the first section we illustrate the identification problem related to interest rate rules and we set out our strategy for empirical investigation. In the second section we discuss our estimation of the structure of the US economy. In the third section, we identify parameters describing central bank's preferences by GMM estimation of the policy maker's Euler equations. We then proceed, in the fourth section, to evaluate optimal and actual interest rate behavior. As the pure inflation targeting model is rejected, we devote the fifth section to analyze Brainard-type uncertainty and real interest rate smoothing as possible extension of the pure model. The sixth section concludes.

## 2. Interest rate rules and central banks' preferences

The successful work by Taylor(1993) has revived interest in the estimation of central banks' reaction functions. The very simple, myopic, framework used in the original paper has been extended to the GMM estimation of forward looking interest rate rules (Clarida, Gali and Gertler,1997, 1998) where interest rate are modelled as a function of lagged interest rates, the gap between the expected rate of inflation and the target rate of inflation and the gap between output and full capacity output. What is surprising is that the literature has used the GMM methodology to estimate reaction functions while the optimization problem of the Central banks provides first order conditions which are instead the natural object of GMM estimation (see Mankiw, Rothemberg and Summers,1985, for a general discussion of this point) . We propose an alternative empirical route to the direct estimation of interest rate rules, which is based on the estimation of a small structural model for the economy and of the Euler equations for the solution of the central banks' optimisation problem. Our strategy allows the identification of the parameters describing central banks' preferences. Given the identification of the structure of the economy and of the monetary policy maker's preferences, we generate interest rates from the optimal rule and compare them with observed policy rates to make inference on the best model to rationalize actual behavior by the Central Bank.

Our point can be made considering the simplest possible version of the inflation targeting problem.

The central bank faces the following intertemporal problem:

$$E_t \sum_{i=0}^{\infty} \delta^i L_{t+i}$$

$$L = \frac{1}{2}\left[(\pi_t - \pi^*)^2 + \lambda x_t^2\right] \tag{2.1}$$

where $E_t$ denotes expectations conditional upon information set available at time t , $\delta$ is the discount factor applied by the central bank, $\pi_t$ is inflation at time t , $\pi^*$ is the target level of inflation, $\lambda$ is a parameter which determines the degree of flexibility in inflation targeting, when $\lambda = 0$ the Central Bank is best described as a strict inflation targeter. As the monetary instrument is the policy rate, $i_t$, the structure of the economy must be described to obtain an explicit form for the policy rule. We consider the following specification for aggregate supply and demand in a closed economy[2]:

$$x_{t+1} = \beta_x x_t - \beta_r \left(i_t - E_t\pi_{t+1} - \bar{r}\right) + u_{t+1}^d \tag{2.2}$$

$$\pi_{t+1} = \pi_t + \alpha_x x_t + u_{t+1}^s \tag{2.3}$$

where $x$ represents deviations of output from its natural level and $\pi$ is the rate of inflation.

As shown in Svensson(1997), the first order conditions for optimality can then be written as follows:

$$\frac{dL}{di_t} = (E_t\pi_{t+2} - \pi^*) = -\frac{\lambda}{\delta\alpha_x k} E_t x_{t+1} \tag{2.4}$$

$$k = 1 + \frac{\delta\lambda k}{\lambda + \delta\alpha_x^2 k} \tag{2.5}$$

By using (2.3) in (2.2) we have:

$$E_t\pi_{t+2} = E_t\pi_{t+1} + \alpha_x \left[\beta_x x_t - \beta_r \left(i_t - E_t\pi_{t+1} - \bar{r}\right)\right] \tag{2.6}$$

[2] Alternatively the supply equation could be reformulated to include a forward looking term in inflation expectations. Moreover in an open-economy both demand and supply equations would include terms in the real exchange rate and foreign prices. This additions would imply new channels of monetary policy transmission to our problem, without altering its fundamental strucutre.

An interest rate rule can be then derived by substituting (2.6)in (2.4):

$$\begin{aligned} i_t &= \bar{r} + \pi^* + \left(\frac{1+\alpha_x\beta_r}{\alpha_x\beta_r}\right)(E_t\pi_{t+1} - \pi^*) + \\ &+ \frac{\beta_x}{\beta_r}x_t + \frac{\lambda}{\delta\alpha_x k}\frac{1}{\alpha_x\beta_r}E_t x_{t+1} \end{aligned} \tag{2.7}$$

A number of comments on the rule are in order:

- If the rule is estimated as a single equation, then the fitted parameters are convolutions of the parameters describing central banks preferences$(\pi^*, \lambda, \delta)$ and the parameters describing the structure of the economy $(\alpha_x, \beta_r, \beta_x, \bar{r})$. In other words estimated interest rate rules are subject to the Lucas (1976) critique

- As the structure of the economy cannot be identified from the estimation of the rule only, it is impossible to assess if the responses of central banks to output and inflation are consistent with the parameters describing the impact of the policy instrument on these variables. Note, for example, that the estimation of an interest rate rule relating the policy rate to the output gap and deviation of expected inflation from target does not allow the identification of a strict inflation targeter($\lambda = 0$ in the terminology of Svensson), from a flexible inflation targeter($\lambda > 0$)

- Econometric identification of the rule requires the timing assumption that the central bank can respond to contemporaneous variables in the economy when setting policy rates, but the policy rates do not have a contemporaneous impact on the macroeconomic variables. This assumption is commonly used to identify VAR models of the monetary transmission mechanism in the US.

- In order to make (2.7) consistent with the data, the rule has been interpreted as delivering desired interest rates and a sluggish adjustment of actual to desired policy rate has been imposed. Direct estimation of the policy rule does not allow to identify the structure of the central bank's preferences which is consistent with interest rate smoothing.

- There is only one empirical implication of the rule which can be confronted with the data independently from the identification of the parameters of interest, namely whether the parameter describing the reaction of policy rates to a gap between current and target inflation is larger than one. In fact, a monetary policy which accommodates changes in inflation generates self-fulfilling bursts of inflation while only if the target real rates adjust to stabilize inflation convergence is achieved at the target rate $\pi^*$. This empirical prediction is the one which has attracted most of the discussion of estimated monetary policy rules (see, for example, Clarida,Gali and Gertler,1997).

In order to provide a better mapping from central banks' behavior to central banks' preferences we propose the following alternative strategy.

First, estimate the structure of the economy to identify the parameters in the aggregate supply and demand functions. Second, estimate the Euler Equation for the solution of the intertemporal problem to identify Central Banks preferences. Note here that in our simple example, given the knowledge of $\alpha_x, \beta_r$, we can identify the $\pi^*$ associated to each $\delta$ from the estimation of the f.o.c(2.4). Third, assess if the monetary policy rule consistent with the structure of the economy and central bank's preferences can successfully describe the actual behavior of policy rates. In other words, estimate a monetary policy rules with the theory-driven specification and test restrictions on identified parameters.

## 3. A stylized representation of the structure of the US economy

The first step of our empirical strategy is the estimation of a small structural model for the US economy, to derive the empirical counterpart of equations (2.3)-(2.2) in our stylized example of the previous section. We concentrate on quarterly data for the period 1960-1998. The empirical literature on the US monetary transmission mechanism has recently reached a consensus(Christiano et al.,1998) on the minimal set of variables needed to describe congruently the US aggregate demand and supply for the analysis of monetary policy. Following this lead, we concentrate on three macroeconomic variables: an indicator of output, inflation and the commodity price index. We consider the output gap as our indicator for output; we define output gap as the percentage difference between GDP and the Hodrick-Prescott filtered series for GDP with smoothing parameter set to 1600. Such choice is in line with the approach followed in the recent empirical literature on monetary policy rules, with which we want to directly compare our results.[3] Inflation is measured by annual CPI inflation. Our commodity price index is the IMF index for all commodities, which has been extensively used as a leading indicator for inflation in the recent VAR literature on the monetary transmission mechanism to obtain a solution of the "price puzzle".

Our baseline specification is a VAR for these three variables. Lags of the policy rate are included as exogenous variables. [4] Such augmented VAR is statistical model on which our small structural model for the US economy is based. We report diagnostic statistics on the reduced form in Table 1.

**Insert Table 1 here**

We note that all the usual diagnostic tests deliver satisfactory results, after the introduction of dummies to take care of outliers mostly driven by shocks in the commodity price index. From our reduced form, we identify a small structural model for the US economy. The full specification is described in Table 2.

**Insert Table 2 here**

The structure of the estimated model, without dummies, is as follows

$$\begin{aligned} x_t &= \beta_1 x_{t-1} - \beta_2 x_{t-3} - \beta_3 \left( i_{t-2} - E_{t-2}\pi_{t-1} - \bar{r} \right) + \beta_4 \left( i_{t-3} - E_{t-3}\pi_{t-2} - \bar{r} \right) + \\ &\quad + u_t^d \qquad (3.1) \\ \pi_t &= \alpha_0 + \alpha_1 \pi_{t-1} - \alpha_2 \pi_{t-4} + \alpha_3 \pi_{t-5} + \alpha_4 x_{t-1} + \alpha_5 \Delta_4 lpcm_t + u_t^s \qquad (3.2) \\ \Delta_4 lpcm_t &= \gamma_0 + \gamma_1 \Delta_4 lpcm_{t-1} - \gamma_2 \Delta_4 lpcm_{t-2} + u_t^c \qquad (3.3) \end{aligned}$$

Where all parameters are positive. We take care of the presence of expectations by estimating the system simultaneously and by imposing the appropriate cross-equation restrictions:

$$E_{t-j-1}\pi_{t-j} = \alpha_0 + \alpha_1 \pi_{t-j-1} - \alpha_2 \pi_{t-j-4} + \alpha_3 \pi_{t-j-5} + \alpha_4 x_{t-j-1} + \alpha_5 \Delta_4 lpcm_{t-j}$$

[3] In fact, quadratic deterministic trends are often used as an alternative to HP filtering. We have made experiments with these two alternative measures for the cycle and found that they deliver very similar results.

[4] The contemporaneous policy rate is omitted, following the conventional approach to identification (see Christiano et al.(1998)).

for j=1,2.

Note that we have asserted the validity of the specification of the reduced form, but the structural model (3.1)-(3.3) imposes further restrictions, which are testable by comparing the unrestricted reduced form with the restricted reduced form implicit in our structural model. On the basis of the outcome of the test[5] we conclude that the rather standard structure of the model (3.1)-(3.3) is not inconsistent with the data, although there might be some room for improving on our specification. [6]

Equation (3.1) identifies aggregate demand. We note the importance of persistence of the effect of real interest rates on demand. As we show in the next section, this justifies an autoregressive structure in the interest rate rule, even when the central bank does not care for interest rates smoothing . We note also that from the estimation of the constant in the aggregate demand we can identify the equilibrium long-term interest rate, in fact $\bar{r}= \frac{\beta_0}{\beta_3-\beta_4}$.Our estimates of the parameters imply an equilibrium real rate around two per cent.

Equation (3.2)identifies aggregate supply. We note that commodity price inflation is a leading indicator of inflation and that also the lagged output gap is significant. Taking into account the dynamic structure of aggregate demand and supply, we observe that it takes about nine months for monetary policy to have a first impact on inflation. Lastly, from (3.3) we note that commodity price inflation is strongly exogenous for the estimation of the parameters of interest in the aggregate demand and supply equations, which means that there is no significant feedback from real monetary policy rates in the US and commodity price inflation. Our estimates for the parameters in the model are very close to those obtained by Rudebusch-Svensson(1998), who adopt a similar specification on a slightly different sample. Hence their comparison with other empirical estimates establishing plausibility and conformity to central banks model applies also our

[5] As shown by Hendry(1996), the relevant test statistic is distributed as a $\chi^2$ with a number of degrees of freedom equal to the number of overidentifying restrictions in the structural model. In our case we have 42 over-identifying restrictions and the observed value for the $\chi^2$ is 78.01, with a tail probability of 0.006 However when we correct for the small sample size as suggested by Sims(1981), the value of the statisitic is reduced to 37.51, with a tail probability of 0.68 . So, the restrictions are not rejected when the correction is implemented but they are without correction.

[6] By comparing the reduced form with the structural model we note that the statistic for the validity of the restrictions increases definitely when the impact of the interest rates on the supply function is set to zero. The possibility of allowing for a direct effect of monetary policy on the supply side is an interesting one, which we plan to consider in future work.

results. We conclude this section by noting the impact effect of monetary policy on inflation, measured by -$\beta_3\alpha_4$ is small, and, given the persistence in the output gap and inflation, it takes a rather long-time for monetary policy to achieve its long-run impact of $\frac{-(\beta_3-\beta_4)\alpha_4}{(1-\beta_1+\beta_2)(1-\alpha_1+\alpha_2-\alpha_3)}$. Our point estimate for the short-run effect is -0.01, while our point estimate for the long-run effect is approximately -0.47.

# 4. Identifying Central Bank preferences

Identification of central bank preferences is achieved by specifying a loss function and by minimizing it intertemporally with respect to the monetary policy instrument subject to the constraint given by the structure of the economy. We consider in turn strict inflation targeting, flexible inflation targeting and the combination of inflation targeting with real interest rate smoothing.

## 4.1. Strict Inflation targeting

Central bank's preferences are described by the following intertemporal loss function:

$$E_t \sum_{i=0}^{\tau} \delta^i L_{t+i} \tag{4.1}$$

where $E_t$ denotes expectations conditional upon information set available at time t , $\delta$ is the discount factor applied by the central bank and the loss function L is specified as follows:

$$L = \frac{1}{2}\left[(\pi_t - \pi^*)^2\right] \tag{4.2}$$

where $\pi_t$ is inflation at time t , $\pi^*$ is the target level of inflation, $x_t$ is the output gap, $\lambda$ is a weight which determines the degree of flexibility in inflation targeting. The central bank proceeds to the minimization under the constraints given by (3.1)-(3.3).

The first order conditions for optimality can then be written as follows:

$$\sum_{i=0}^{\tau} \delta^i E_{t+j}\left(\pi_{t+i+j} - \pi^*\right)\frac{\partial \pi_{t+i+j}}{\partial i_{t+j}} = 0 \qquad j = 1, ..., \tau \tag{4.3}$$

where, due to the linearity of the structure of the economy $\frac{\partial \pi_{t+i+j}}{\partial i_{t+j}}$ are constant. Svensson(1996) notes that there is a simple strategy for satisfying the first order conditions for optimality. In fact, the structure of the economy is such that $\pi_{t+i+3}$ can be controlled by $i_{t+i}$ and it is not affected by $i_{t+i+1}$, $i_{t+i+2}$...., so each $i_{t+i}$ can be chosen such that $E_{t+j}\left(\pi_{t+i+j}-\pi^{*}\right)=0$. Due to the law of iterated expectations it follows that $E_{t+j}\left(\pi_{t+i+j}-\pi^{*}\right)=0$, for j>3, and the first order conditions for optimality (**??**)can then be re-written simply as follows:

$$E_{t}\left(\pi_{t+3}-\pi^{*}\right)=0 \tag{4.4}$$

Note that (4.4) is the natural object to be estimated by GMM, although in this simple case the estimation would not be very informative, in that it would just set $\pi^{*}$ to the sample mean of inflation. (4.4) makes also clear that strict inflation targeting implies an extremely aggressive monetary policy, capable of cancelling from the dynamic of inflation expectations the stickiness of the past inflationary process. We use this observation and test strict inflation targeting by deriving the optimal interest rate rule and comparing it with the actual behavior of interest rates.

To derive the optimal interest rate rule consider that .

$$E_{t}\pi_{t+3}=\alpha_{0}+\alpha_{1}E_{t}\pi_{t+2}+\alpha_{2}\pi_{t-1}+\alpha_{3}\pi_{t-2}+\alpha_{4}E_{t}x_{t+2}+\alpha_{5}E_{t}\Delta_{4}lpcm_{t+3} \tag{4.5}$$

$$E_{t}x_{t+2}=\beta_{1}E_{t}x_{t+1}-\beta_{2}x_{t-1}-\beta_{3}\left(i_{t}-4E_{t}\pi_{t+1}-\bar{r}\right)+\beta_{4}\left(i_{t-1}-4E_{t-1}\pi_{t}-\bar{r}\right) \tag{4.6}$$

Now, by using (4.5) and (4.6) in (4.4), we have

$$\begin{aligned} \pi^{*} \;=\;& \alpha_{0}+\alpha_{1}E_{t}\pi_{t+2}+\alpha_{2}\pi_{t-1}+\alpha_{3}\pi_{t-2}+\alpha_{5}E_{t}\Delta_{4}lpcm_{t+3}+ \\ & \alpha_{4}\beta_{1}E_{t}x_{t+1}-\alpha_{4}\beta_{2}x_{t-1}+ \\ & -\alpha_{4}\beta_{3}\left(i_{t}-E_{t}\pi_{t+1}-\bar{r}\right)+\alpha_{4}\beta_{4}\left(i_{t-1}-4E_{t-1}\pi_{t}-\bar{r}\right) \end{aligned} \tag{4.7}$$

From which a quasi-standard forward-looking interest rate rule can be derived as:

$$\left(i_{t}-E_{t}\pi_{t+1}-\bar{r}\right) \;=\; \frac{\beta_{4}}{\beta_{3}}\left(i_{t-1}-E_{t-1}\pi_{t}-\bar{r}\right)+ \tag{4.8}$$

$$+\frac{1}{\beta_3\alpha_4}\left(\alpha_0+\alpha_1 E_t\pi_{t+2}+\alpha_2\pi_{t-1}+\alpha_3\pi_{t-2}-\pi^*\right)+$$
$$+\left(\beta_1 E_t x_{t+1}-\beta_2 x_{t-1}\right)\frac{1}{\beta_3}+\frac{\alpha_5}{\beta_3\alpha_4}E_t\Delta_4 lpcm_{t+3}$$

Note that the optimal rule features interest rate persistence even if interest rate smoothing does not enter in the central banks' preferences. Such persistence is entirely due to the dynamic structure of the economy. This is not a new result (see Sack,1998). The parameters estimated from the structural model illustrate how aggressive is the optimal monetary policy: think of the parameters with which the central Bank should react to deviation of inflation from target:$\frac{1}{\beta_3\alpha_4}$. As our point estimates of $\beta_3$ and $\alpha_4$ are respectively of 0.12 and 0.11, the optimal reaction features a parameter on $(\alpha_0+\alpha_1 E_t\pi_{t+2}+\alpha_2\pi_{t-1}+\alpha_3\pi_{t-2}-\pi^*)$ of about one hundred. To compare actual with optimal interest rate behavior and to have some evidence on the sources of divergences between the two series we estimate by GMM the following model:

$$\begin{aligned}\left(i_{t-1}-E_{t-1}\pi_t-\bar{r}\right) &= c_1\left(i_t-E_t\pi_{t+1}-\bar{r}\right)+ \\ & c_2\left(\alpha_0+\alpha_1 E_t\pi_{t+2}+\alpha_2\pi_{t-1}+\alpha_3\pi_{t-2}-\pi^*\right)+ \\ & c_3\left(\beta_1 E_t x_{t+1}-\beta_2 x_{t-1}\right)+c_4 E_t\Delta_4 lpcm_{t+3}\end{aligned} \tag{4.9}$$

If actual and optimal interest rate behavior do no diverge then (i)(4.9) should deliver a congruent specification for the interest rate (ii) the following restrictions should not be rejected

$$c_1=\frac{\beta_3}{\beta_4}, c_2=-\frac{1}{\beta_4\alpha_4}, c_3=-\frac{1}{\beta_4}, c_4=-\frac{\alpha_5}{\beta_4\alpha_4}$$

We have estimated our equation from 1983:1 onwards, since it is clear that the Fed has not always followed an interest rate rule prior to 1983. We note that the we have used data after 1983:1 for the estimation of the structure of the US economy. However, the estimates of structural parameters of interest are rather stable and they are not different if obtained using the sample 1960-1983 instead of 1960-1998. We adopt an estimation method appropriate to take account of the existence of an MA(3) in the residuals, generated by having four-periods ahead expectations in the first order conditions. The results reported in Table 3 clearly illustrate that while (4.9) is broadly consistent with the data, the above restrictions are overwhelmingly rejected.

**Insert Table 3 here**

We can then conclude that the strict inflation targeting model pins down correctly the variables to which the Fed reacts, but does not have the right prediction of the coefficients determining the magnitude of the responses. In fact, Fed's response to the relevant variables is much smoother than the one predicted by the strict inflation targeting model. In the next sections we use our methodology to evaluate alternative potential explanations of such empirical evidence.

### 4.2. Flexible inflation targeting

Within a flexible inflation targeting framework, given the rather complicated dynamics of the economy, the solution of the intertemporal optimization problem with infinite horizon would generate an extremely complicated Euler equation with many collinear terms. To obtain a manageable solution we simplify the problem and consider an horizon of one-year.[7] Central bank's preferences are described by the following intertemporal loss function:

$$E_t \sum_{i=0}^{\tau} \delta^i L_{t+i} \tag{4.10}$$

where $E_t$ denotes expectations conditional upon information set available at time t , $\delta$ is the discount factor applied by the central bank and the loss function L is specified as follows:

$$L = \frac{1}{2}\left[(\pi_t - \pi^*)^2 + \lambda x_t^2\right] \tag{4.11}$$

where $\pi_t$ is inflation at time t , $\pi^*$ is the target level of inflation, $x_t$ is the output gap, $\lambda$ is a weight which determines the degree of flexibility in inflation targeting. The central bank proceeds to the minimization under the constraints given by (3.1)-(3.3).

The first order conditions for optimality for current decision variables can then be written as follows:

[7] The robustness of our choice of the horizon can be checked by extending it by one period, and testing if the additional variables involved in the Euler equation attract significant coefficients. We have done so by setting $\tau = 5$, in all our empirical applications and found support for the choice $\tau = 4$.

$$E_t \sum_{i=0}^{\tau} \delta^i E_t (\pi_{t+i} - \pi^*) \frac{\partial \pi_{t+i}}{\partial i_t} + E_t \sum_{i=0}^{\tau} \delta^i \lambda E_t x_{t+i} \frac{\partial x_{t+i}}{\partial i_t} = 0 \tag{4.12}$$

For a one-year horizon($\tau = 4$) , the conditions for optimality can be written as follows:

$$\begin{aligned}
&\lambda E_t x_{t+2} \frac{\partial x_{t+2}}{\partial i_t} + \delta \lambda E_t x_{t+3} \left( \frac{\partial x_{t+3}}{\partial i_t} + \frac{\partial x_{t+3}}{\partial x_{t+2}} \frac{\partial x_{t+2}}{\partial i_t} \right) + \qquad (4.13)\\
&\delta^2 \lambda E_t x_{t+4} \frac{\partial x_{t+4}}{\partial x_{t+3}} \left( \frac{\partial x_{t+3}}{\partial i_t} + \frac{\partial x_{t+3}}{\partial x_{t+2}} \frac{\partial x_{t+2}}{\partial i_t} \right) + \\
&\delta E_t (\pi_{t+3} - \pi^*) \left( \frac{\partial \pi_{t+3}}{\partial x_{t+2}} \frac{\partial x_{t+2}}{\partial i_t} \right) + \\
&\delta^2 E_t (\pi_{t+4} - \pi^*) \left( \frac{\partial \pi_{t+4}}{\partial x_{t+3}} \left( \frac{\partial x_{t+3}}{\partial i_t} + \frac{\partial x_{t+3}}{\partial x_{t+2}} \frac{\partial x_{t+2}}{\partial i_t} \right) + \frac{\partial \pi_{t+4}}{\partial \pi_{t+3}} \left( \frac{\partial \pi_{t+3}}{\partial x_{t+2}} \frac{\partial x_{t+2}}{\partial i_t} \right) \right) \\
= &\ 0 \qquad (4.14)
\end{aligned}$$

and, substituting for the estimated parameters

$$\begin{aligned}
&-\lambda E_t x_{t+2} \beta_3 + \delta \lambda E_t x_{t+3} (\beta_4 - \beta_1 \beta_3) + \delta^2 \lambda E_t x_{t+4} \beta_1 (\beta_4 - \beta_1 \beta_3) \qquad (4.15)\\
&-\delta E_t (\pi_{t+3} - \pi^*) \alpha_4 \beta_3 + \delta^2 E_t (\pi_{t+4} - \pi^*) \alpha_4 (\beta_4 - \beta_3 (1 + \beta_1)) \\
= &\ 0
\end{aligned}$$

Equation (4.15)is an orthogonality condition, which can be exploited to estimate by GMM the parameters describing central bank's preferences. Such parameters are identifiable as the $\alpha' s$ and $\beta' s$ parameters describing the structure of the economy are separately identified in the small model estimated in the previous section.

We thus estimate the condition for optimality in the following format:

$$\begin{aligned}
&E_t \pi_{t+3} = \pi^* + \delta E_t (\pi_{t+4} - \pi^*) \left( \frac{\beta_4 - \beta_3 (1 + \beta_1)}{\beta_3} \right) + \qquad (4.16)\\
&-\lambda \left( E_t x_{t+2} \frac{1}{\alpha_4 \delta} - E_t x_{t+3} \frac{(\beta_4 - \beta_1 \beta_3)}{\alpha_4 \beta_3} - \delta E_t x_{t+4} \frac{\beta_1 (\beta_4 - \beta_1 \beta_3)}{\alpha_4 \beta_3} \right)
\end{aligned}$$

Equation (4.16)embodies testable parameters restrictions implied by our estimates of the demand and supply system. Results are reported in Table 4.

**Insert Table 4 here**

Following, a common practice in the empirical literature on the simulation of interest rates rules we fix $\delta$ at .975, which implies a discount rate of 2.5 per cent.[8] Given that we restrict the $\alpha's$ and $\beta's$ parameters to the values obtained in the structural model, we are left with only two preferences parameters to be estimated: $\pi^*$ and $\lambda$. However, consistency of our estimates is conditional upon the validity of the theoretical model, which can be tested independently from the identification of $\pi^*$ and $\lambda$. Consider the coefficient on the deviation of expected inflation at time t+4: the restriction $\left(\frac{\beta_4}{\beta_3} - \alpha_1\right) = c_1$ is testable independently from the identification of $\pi^*$ and $\lambda$. Looking at the estimates of the unrestricted model we immediately see that the Euler equation derived from flexible inflation targeting is not consistent with our data-set, in fact the point estimate of $c_1$ is as high as 0.87 while estimates of $c_2, c_3$, and $c_4$,although only marginally significant, have a different sign form that predicted by the theoretical model. When formally tested, the restriction $\left(\frac{\beta_4}{\beta_3} - \alpha_1\right) = c_1$, is clearly rejected. In fact, the value of $c_1$ at the lower limit of the 95 per cent confidence interval is 0.74. Which tells us that the actual interest rate policy is far less aggressive than the optimal interest rate policy. Such excessive smoothness of the policy rates generates a persistence in expected inflation which violates flexible and strict inflation targeting for our parameterization of the US economy.

## 4.3. Real interest rate smoothing

We consider the following modification of central banks' preferences:

$$L = \frac{1}{2}\left[(\pi_t - \pi^*)^2 + \mu\left[(i_t - E_t\pi_{t+1} - r) - (i_{t-1} - E_{t-1}\pi_t - r)\right]^2 + \lambda x_t^2\right]$$

Keeping the structure of the economy is unaltered, the first order conditions for the solution of the central banks' intertemporal optimization problem can now be written as follows:

$$\begin{aligned} &-\delta^2\lambda E_t x_{t+2}\beta_3 + \delta^3\lambda E_t x_{t+3}(\beta_4 - \beta_1\beta_3) + \delta^4\lambda E_t x_{t+4}\beta_1(\beta_4 - \beta_1\beta_3) + \\ &-\delta^3 E_t(\pi_{t+3} - \pi^*)\alpha_4\beta_3 + \delta^4 E_t(\pi_{t+4} - \pi^*)\alpha_4(\beta_4 - \beta_3(1 + \beta_1)) + \end{aligned} \qquad (4.17)$$

[8] When such coefficient is left unrestricted, we obtain a negative point estimate, altough hardly significant.

$$
\begin{aligned}
&+\mu\left(1+\delta\right)\left(i_t - E_t\pi_{t+1} - \bar{r}\right) - \mu\left(i_{t-1} - E_{t-1}\pi_t - r\right) - \mu\delta\left(E_t i_{t+1} - E_t\pi_{t+2} - \bar{r}\right) \\
= \;& 0
\end{aligned}
$$

Which, for the sake of estimation, can be written as:

$$
\begin{aligned}
\left(i_{t-1} - E_{t-1}\pi_t - \bar{r}\right) \;=\;& \left(1+\delta\right)\left(i_t - E_t\pi_{t+1} - \bar{r}\right) - \delta\left(E_t i_{t+1} - E_t\pi_{t+2} - \bar{r}\right) \\
& -\frac{\delta^3}{\mu}E_t\left(\pi_{t+3} - \pi^*\right)\alpha_4\beta_3 + \frac{\delta^4}{\mu}E_t\left(\pi_{t+4} - \pi^*\right)\alpha_4\left(\beta_4 - \beta_3\left(1+\beta_1\right)\right) \quad (4.18) \\
& -\frac{\delta^2}{\mu}\lambda E_t x_{t+2}\beta_3 + \frac{\delta^3}{\mu}\lambda E_t x_{t+3}\left(\beta_4 - \beta_1\beta_3\right) + \frac{\delta^4}{\mu}\lambda E_t x_{t+4}\beta_1\left(\beta_4 - \beta_1\beta_3\right)
\end{aligned}
$$

The results of our estimation are reported in Table 5.

**Insert Table 5 here**

We obtain estimates of three parameters: a constant, which is a convolution of $\bar{r}$ and $\pi^*$, $\frac{1}{\mu}$ and $\lambda$. Since the latter is not significantly different from zero, we reject the hypothesis of flexible inflation targeting and we re-estimate the equation by imposing $\lambda = 0$. The results are reported in Table 6, from where we obtain a significant point estimate for $\mu$ of 0.286, with a 95 per cent confidence interval ranging from 0.10 to 0.47.

**Insert Table 6 here**

The ability of the model to track actual interest rates is shown in Figure 1

**Insert Figure 1 here**

## 5. Conclusions

In this paper we develop an approach to identify central banks' preferences, which differs from the standard practice of estimating unrestricted (forward-looking) interest rate rules. Since estimated parameters in a monetary policy rule are convolutions of "deep" parameters describing central banks' preferences and those describing the structure of the economy, it is not possible to identify central banks' preferences from the direct estimation of monetary policy rules. However, such preferences can be naturally identified from the first order conditions of the central banks' intertemporal optimization problem for a given structure of the economy.

We have applied our approach to the US, taken as an example of a closed economy. We first estimated the parameters describing the structure of the economy by considering a parsimonious specification for inflation, the output-gap and the commodity price index. We then proceeded to the identification of the Fed preferences by estimating by GMM the Euler equations obtained from the first order conditions for the solution to the problem relevant to the central banker. We have considered in turn strict inflation targeting, flexible inflation targeting and inflation targeting with real interest rate smoothing.

Our empirical analysis of the structure of the economy has shown that the persistency of the effect of real interest rates on aggregate demand is sufficient to generate an autoregressive structure in any interest rate rule. From the estimation of Euler equations, we infer that strict inflation targeting together with real interest rate smoothing delivers an optimal policy rule capable of replicating the observed path of real interest rates over the sample 1983:1-1998:3. Our empirical findings imply that the output gap enters into the optimal interest rate rule only as a leading indicator of future inflation, and we reject the hypothesis that output stabilization is an independent argument in the loss function of the Fed.

We believe that direct estimation of the Euler equation is the appropriate approach to the identification of central bank preferences. Hopefully the framework used in this paper can be extended to accommodate more complex structural models and alternative hypotheses on central banks' behavior.

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| Table 1: Diagnostic statistics on the unrestricted system | | | |
|---|---|---|---|
| | Autocorrelation | Normality | Het |
| Eq 1 | AR 1-5 F( 5,124) = 1.0296 [0.4033] | $\text{Chi}^2$(2)= 5.0107 [0.0816] | ARCH 4 F( 4,121) = 0.6835 [0.6047] $\text{Xi}^2$ F(38, 90) = 1.5493 [0.0471] |
| Eq 2 | AR 1-5 F( 5,124) = 2.0249 [0.0796] | $\text{Chi}^2$(2)= 2.7121 [0.2577] | ARCH 4 F( 4,121) = 0.6835 [0.6047] $\text{Xi}^2$ F(38, 90) = 1.5493 [0.0471] |
| Eq 3 | AR 1-5 F( 5,124) = 1.2414 [0.2938] | $\text{Chi}^2$(2)= 6.497 [0.0388] | ARCH 4 F( 4,121) = 0.6835 [0.6047] $\text{Xi}^2$ F(38, 90) = 1.5493 [0.0471] |
| System | AR 1-5 F(45,333) = 1.3174 [0.0926] | $\text{Chi}^2$( 6)= 10.157 [0.1182] | $\text{Xi}^2$ F(228,513) = 1.2519 [0.0209] |

The statistic for residual autocorrelation is the LM statistic for lags 1 to 5, the two statistics for heterescedasticity test The null of homoscedastic residuals against the alternative of ARCH and heteroscedasticity due to the squares of regressors and their cross products. All the test are performed both separately on each equation and on the system as a whole. For a detailed description of these tests see Doornik-Hendry(1997). The estimated system is :

$$\begin{pmatrix} x_t \\ \pi_t \\ \Delta_4 lcpm_t \end{pmatrix} = A(L) \begin{pmatrix} x_{t-1} \\ \pi_{t-1} \\ \Delta_4 lcpm_{t-1} \end{pmatrix} + B(L) i_{t-1} + C \begin{pmatrix} oil73 \\ oil75 \\ oil783 \\ dum862 \\ dum773 \end{pmatrix} + \begin{pmatrix} u_t^d \\ u_t^s \\ u_t^c \end{pmatrix}$$

$x_t$ = output gap (difference between real GDP and HP filtered GDP with smoothing parametr set at 1600), $\pi_t$ = CPI annual inflation rate, $\Delta_4 lcpm_t$ = IMF commodity price index (annual growth rate), $i_t$ = Federal funds rate (average value in the last month of each quarter), $dum7323$ = dummy taking a value of 1 in 1973:1 and 1973:2, zero otherwise, $dum751$ = dummy taking a value of 1 in 1975:1, zero otherwise, $dum773$ = dummy taking a value of 1 in 1977:3, zero otherwise, $dum783$ = dummy taking a value of 1 in 1978:1, zero otherwise, $dum862$ = dummy taking a value of 1 in 1986:2, zero otherwise

| Table 2: A structural model for the US Economy | | | | |
|---|---|---|---|---|
| Estimation Method: SURE,sample 1960:1-1998:3 | | | | |
| | Coefficient | Std. Error | t-Statistic | Prob. |
| $\alpha_0$ | 0.0883 | 0.0690 | 1.2805 | 0.20 |
| $\alpha_1$ | 1.0931 | 0.0353 | 30.9622 | 0.00 |
| -$\alpha_2$ | -0.4166 | 0.0833 | -5.0001 | 0.00 |
| $\alpha_3$ | 0.2940 | 0.0638 | 4.6070 | 0.00 |
| $\alpha_4$ | 0.1221 | 0.0278 | 4.3986 | 0.00 |
| $\alpha_5$ | 0.0160 | 0.0032 | 5.0361 | 0.00 |
| -$\alpha_6$ | -1.3754 | 0.4395 | -3.1297 | 0.002 |
| $\alpha_7$ | 1.0236 | 0.4476 | 2.2868 | 0.023 |
| $\beta_0$ | 0.0437 | 0.0791 | 0.5521 | 0.582 |
| $\beta_1$ | 1.0507 | 0.0468 | 22.4643 | 0.00 |
| -$\beta_2$ | -0.2746 | 0.0453 | -6.0619 | 0.00 |
| -$\beta_3$ | -0.1377 | 0.0399 | -3.4508 | 0.001 |
| $\beta_4$ | 0.1120 | 0.0405 | 2.7622 | 0.006 |
| $\beta_5$ | 3.3098 | 0.6835 | 4.8423 | 0.00 |
| -$\beta_6$ | -1.8100 | 0.6814 | -2.6563 | 0.008 |
| $\gamma_0$ | 0.4118 | 0.3074 | 1.3397 | 0.181 |
| $\gamma_1$ | 1.1541 | 0.0652 | 17.709 | 0.00 |
| -$\gamma_2$ | -0.1723 | 0.0772 | -2.2325 | 0.026 |
| -$\gamma_3$ | -0.4364 | 0.0769 | -5.6721 | 0.000 |
| $\gamma_4$ | 0.2547 | 0.0649 | 3.9215 | 0.000 |
| $\gamma_5$ | 12.356 | 1.9562 | 6.3164 | 0.000 |
| -$\gamma_6$ | -16.727 | 3.8858 | -4.3046 | 0.000 |
| $\gamma_7$ | 7.1929 | 3.7248 | 1.9311 | 0.054 |
| -$\gamma_8$ | -6.7065 | 3.5938 | -1.8661 | 0.063 |
| -$\gamma_9$ | -15.878 | 3.6220 | -4.3839 | 0.000 |

$$
\begin{aligned}
\pi_t &= \alpha_0 + \alpha_1 \pi_{t-1} - \alpha_2 \pi_{t-4} + \alpha_3 \pi_{t-5} + \alpha_4 x_{t-1} + \alpha_5 \Delta_4 lpcm_t - \\
&\quad -\alpha_6 dum862 + \alpha_7 dum782 + u_t^s \\
x_t &= \beta_1 x_{t-1} - \beta_2 x_{t-3} - \beta_3 \left( i_{t-2} - E_{t-2}\pi_{t-1} - \bar{r} \right) + \beta_4 \left( i_{t-3} - E_{t-3}\pi_{t-2} - \bar{r} \right) \\
&\quad +\beta_5 dum782 - \beta_6 dum751 + u_t^d \\
\Delta_4 lpcm_t &= \gamma_0 + \gamma_1 \Delta_4 lpcm_{t-1} - \gamma_2 \Delta_4 lpcm_{t-2} - \gamma_3 \Delta_4 lpcm_{t-4} + \gamma_4 \Delta_4 lpcm_{t-5} \\
&\quad +\gamma_5 dum7323 - \gamma_6 dum751 + +\gamma_7 dum783 + -\gamma_8 dum862 - \gamma_9 dum773 + u_t^c
\end{aligned}
$$

where

$$E_{t-2}\pi_{t-1} = \alpha_0 + \alpha_1\pi_{t-2} - \alpha_2\pi_{t-3} + \alpha_3\pi_{t-4} + \alpha_4 x_{t-2} + \alpha_5\Delta_4 lpcm_{t-1} - -\alpha_6 dum862_{t-1} + \alpha_7 dum782_{t-1}$$

and similarly for $E_{t-3}\pi_{t-2}$

**Table 2 continued: Diagnostics on the structural equations:**

| | | |
|---|---|---|
| $\pi_t = \alpha_0 + \alpha_1\pi_{t-1} - \alpha_2\pi_{t-4} + \alpha_3\pi_{t-5} + \alpha_4 x_{t-1} + \alpha_5\Delta_4 lpcm_t$ | | |
| $-\alpha_6 dum862 + \alpha_7 dum782 + u_t^s$ | | |
| $R^2$ 0.976 | Adj $R^2$ 0.975 | S.E. of reg 0.456 |
| DW stat 1.987 | Mean dep. var 4.45 | S.D. dep var 2.90 |

| | | |
|---|---|---|
| $x_t = \beta_1 x_{t-1} - \beta_2 x_{t-3} - \beta_3\left(i_{t-2} - E_{t-2}\pi_{t-1} - \bar{r}\right) + \beta_4\left(i_{t-3} - E_{t-3}\pi_{t-2} - \bar{r}\right)$ | | |
| $+\beta_5 dum782 - \beta_6 dum751 + u_t^d$ | | |
| $R^2$ 0.824 | Adj $R^2$ 0.806 | S.E. of reg 0.711 |
| DW stat 2.02 | Mean dep var -0.041 | S.D. dep var 1.615 |

| | | |
|---|---|---|
| $\Delta_4 lpcm_t = \gamma_0 + \gamma_1\Delta_4 lpcm_{t-1} - \gamma_2\Delta_4 lpcm_{t-2} - \gamma_3\Delta_4 lpcm_{t-4} + \gamma_4\Delta_4 lpcm_{t-5}$ | | |
| $+\gamma_5 dum7323 - \gamma_6 dum751 + \gamma_7 dum783 + -\gamma_8 dum862 - \gamma_9 dum773 + u_t^c$ | | |
| $R^2$ 0.923 | Adj $R^2$ 0.918 | S.E. of reg 3.668 |
| DW stat 1.793 | Mean dep var 2.873 | S.D. dep var 12.782 |

**Table 3: Comparing Optimal and Actual Interest Rate Rules**
The estimated interest rate reaction equation has the following specification:

$$\begin{aligned}\left(i_{t-1}-E_{t-1}\pi_t-\bar{r}\right) &= c_0+c_1\left(i_t-E_t\pi_{t+1}-\bar{r}\right)+\\ &\quad c_2\left(\alpha_0+\alpha_1 E_t\pi_{t+2}+\alpha_2\pi_{t-1}+\alpha_3\pi_{t-2}-\pi^*\right)+\\ &\quad c_3\left(\beta_1 E_t x_{t+1}-\beta_2 x_{t-1}\right)+c_4 E_t\Delta_4 lpcm_{t+3}\end{aligned}$$

| | | | | |
|---|---|---|---|---|
| Method: Generalized Method of Moments, Sample(adjusted): 1983:1 1997:3 | | | | |
| Bandwidth: Fixed (4) | | | | |
| Instrument list: $\pi_t$, $\pi_{t-1}$, $\pi_{t-2}$, $\pi_{t-3}$, $\pi_{t-4}$, $x_t$, $x_{t-1}$, $x_{t-2}$, | | | | |
| $\Delta_4 lpcm_t$, $\Delta_4 lpcm_{t-1}$, $\Delta_4 lpcm_{t-2}$, | | | | |
| $\Delta_4 lpcm_{t-3}$, $\Delta_4 lpcm_{t-4}$, $i_t$, $i_{t-1}$ | | | | |
| | Coefficient | Std. Error | t-Statistic | Prob. |
| $c_0$ | 0.352 | 0.142 | 2.51 | 0.015 |
| $c_1$ | 0.936 | 0.034 | 27.55 | 0.0000 |
| $c_2$ | 0.20 | 0.12 | -1.73 | 0.088 |
| $c_3$ | -0.573 | 0.147 | -3.88 | 0.000 |
| $c_4$ | -0.060 | 0.013 | -4.34 | 0.000 |
| | | | | |
| R-squared 0.767 | S.E. of regression 0.911 | | | |
| Mean dependent var 3.26 | J-statistic 0.157 | | | |

The theoretical model is

$$\begin{aligned}\left(i_{t-1}-E_{t-1}\pi_t-\bar{r}\right) &= \frac{\beta_3}{\beta_4}\left(i_t-E_t\pi_{t+1}-\bar{r}\right)+ \qquad (5.1)\\ &\quad -\frac{1}{\beta_4\alpha_4}\left(\alpha_0+\alpha_1 E_t\pi_{t+2}+\alpha_2\pi_{t-1}+\alpha_3\pi_{t-2}-\pi^*\right)+\\ &\quad -\left(\beta_1 E_t x_{t+1}-\beta_2 x_{t-1}\right)\frac{1}{\beta_4}-\frac{\alpha_5}{\beta_4\alpha_4}E_t\Delta_4 lpcm_{t+3}\\ &\quad +\frac{1}{\alpha_4}\frac{\delta}{\beta_3}E_t\left(\pi_{t+4}-\pi^*\right)\end{aligned}$$

the following restrictions are then tested

$$c_1=\frac{\beta_3}{\beta_4}, c_2=-\frac{1}{\beta_4\alpha_4}, c_3=-\frac{1}{\beta_4}, c_4=-\frac{\alpha_5}{\beta_4\alpha_4}, c_5=\frac{1}{\alpha_4}\frac{\delta}{\beta_3}$$

$$c_1 = \frac{0.138}{0.112}, c_2 = -\frac{1}{0.112 * 0.122}, c_3 = -\frac{1}{0.112}, c_4 = -\frac{0.016}{0.112 * 0.122}$$

**Coefficient restrictions Wald tests:**F-statistic 7653719 (0.000000),Chi-square 30614876 (0.000000)

**Table 4: Euler Equation Estimate of the first order conditions for the intertemporal optimization problem generated by flexible inflation targeting**

The estimated equation has the following specification:

$$E_t\pi_{t+3} = c_0 + \delta E_t\left(\pi_{t+4} - c_0\right)c_1 + \\ -E_t x_{t+2}\frac{c_2}{\delta} + E_t x_{t+3}c_3 + \delta E_t x_{t+4}c_4 \quad (5.2)$$

Given that flexible inflation targeting would deliver:

$$E_t\pi_{t+3} = \pi^* + \delta E_t\left(\pi_{t+4} - \pi^*\right)\left(\frac{\beta_4 - \beta_3\left(1+\beta_1\right)}{\beta_3}\right) + \\ -\lambda\left(E_t x_{t+2}\frac{1}{\alpha_4\delta} - E_t x_{t+3}\frac{\left(\beta_4 - \beta_1\beta_3\right)}{\alpha_4\beta_3} - \delta E_t x_{t+4}\frac{\beta_1\left(\beta_4 - \beta_1\beta_3\right)}{\alpha_4\beta_3}\right) \quad (5.3)$$

where the parameters $\alpha's$ and $\beta's$ have been substituted with their fitted values (reported in Table 2) and the value 0.975 is assigned to the parameter $\delta$ .

| | | | | |
|---|---|---|---|---|
| Method: Generalized Method of Moments | | | | |
| Sample(adjusted): 1983:1 1997:3 | | | | |
| Bandwidth: Fixed (3),Kernel: Bartlett | | | | |
| Instrument list: $\pi_t, \pi_{t-1}, \pi_{t-2}, \pi_{t-3}, \pi_{t-4}, \pi_{t-5}, x_t, x_{t-1}, x_{t-2}, \Delta_4 lpcm_t,$ $\Delta_4 lpcm_{t-1}$ , $\Delta_4 lpcm_{t-2}$, $\Delta_4 lpcm_{t-3}$, , $\Delta_4 lpcm_{t-4}, i_t, i_{t-1}, dum862$ | | | | |
| | Coefficient | Std. Error | t-Statistic | Prob. |
| $c_0$ | 4.094 | 0.537 | 7.62 | 0.000 |
| $c_1$ | 0.875 | 0.066 | 13.27 | 0.000 |
| $c_2$ | 0.459 | 0.202 | 2.27 | 0.027 |
| $c_3$ | -0.087 | 0.401 | -0.21 | 0.829 |
| $c_4$ | -0.377 | 0.217 | -1.73 | 0.087 |
| R-squared 0.723, S.E. of regression 0.58 | | | | |
| Mean dependent var 3.341868, J-statistic 0.16 | | | | |

Wald test $\left(\frac{\beta_4-\beta_3(1+\beta_1)}{\beta_3}\right) = c_1$,F-statistic 808, Probability 0.000

**Table 5: Euler Equation Estimate of the first order conditions for the intertemporal optimization problem generated by flexible inflation targeting and real interest rate smoothing**

We proceed to the estimation of the following Euler equation:

$$\begin{aligned}(i_{t-1} - E_{t-1}\pi_t) &= c_0 + (1+\delta)(i_t - E_t\pi_{t+1}) - \delta(E_t i_{t+1} - E_t\pi_{t+2}) \\ &-\frac{\delta^3}{\mu}\alpha_4\beta_3\left[E_t(\pi_{t+3}) - \delta E_t(\pi_{t+4})\left(\frac{\beta_4}{\beta_3} - (1+\beta_1)\right)\right] + \\ &-\frac{\delta^2}{\mu}\lambda\beta_3\left[E_t x_{t+2} - \delta E_t x_{t+3}\left(\frac{\beta_4}{\beta_3} - \beta_1\right) - \delta^2 E_t x_{t+4}\beta_1\left(\frac{\beta_4}{\beta_3} - \beta_1\right)\right]\end{aligned} \quad (5.4)$$

| | | | | |
|---|---|---|---|---|
| Method: Generalized Method of Moments, Sample(adjusted): 1983:1 1997:3 | | | | |
| Bandwidth: Fixed (4) | | | | |
| Instrument list: $\pi_t$, $\pi_{t-1}$, $\pi_{t-2}$, $\pi_{t-3}$, $\pi_{t-4}$, $x_t$, $x_{t-1}$, $x_{t-2}$,$x_{t-3}$, $x_{t-4}$, | | | | |
| $\Delta_4 lpcm_t$, $\Delta_4 lpcm_{t-1}$, $\Delta_4 lpcm_{t-2}$, $\Delta_4 lpcm_{t-3}$, $\Delta_4 lpcm_{t-4}$, $i_t$, $i_{t-1}$ | | | | |
| | Coefficient | Std. Error | t-Statistic | Prob. |
| $c_0$ | 0.4315 | 0.2014 | 2.1425 | 0.036 |
| $\frac{1}{\mu}$ | 3.8251 | 1.983 | 1.9289 | 0.058 |
| $\lambda$ | -0.032 | 0.106 | -0.306 | 0.76 |
| | | | | |
| R-squared 0.736 | S.E. of regression 0.962 | | | |
| Mean dependent var 3.254 | J-statistic 0.17 | | | |

where $\delta$ has been fixed at .975.

**Table 6: Euler Equation Estimate of the first order conditions for the intertemporal optimization problem generated by strict inflation targeting and real interest rate smoothing**

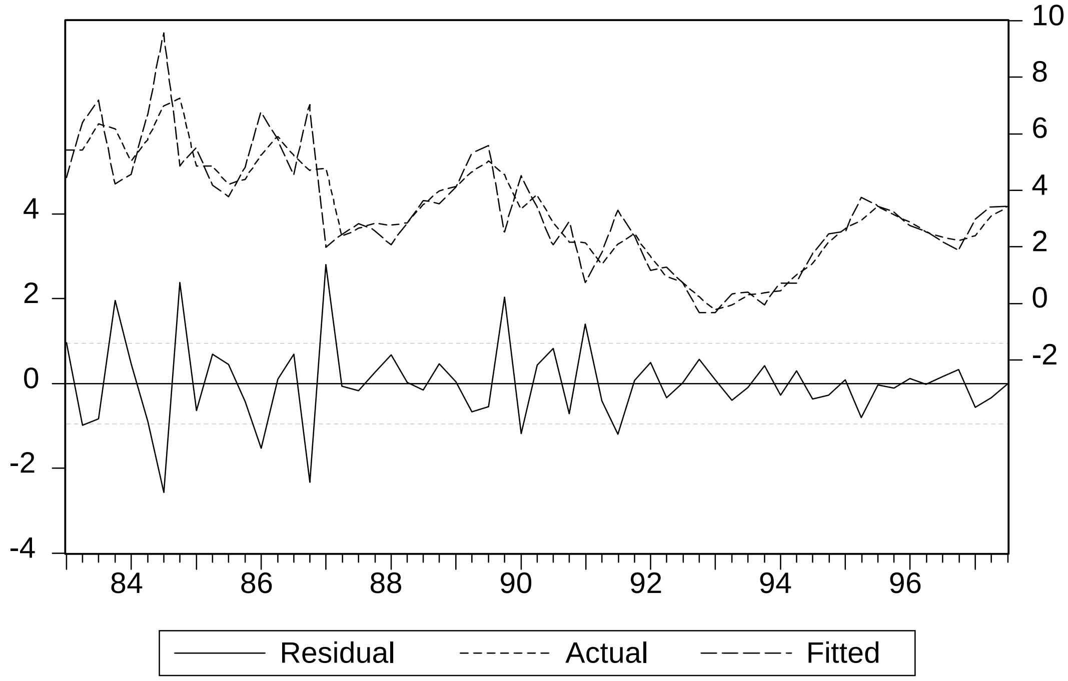
We proceed to the estimation of the following Euler equation:

$$\begin{aligned}(i_{t-1} - E_{t-1}\pi_t) &= c_0 + (1+\delta)(i_t - E_t\pi_{t+1}) - \delta(E_t i_{t+1} - E_t\pi_{t+2}) \\ &\quad - \frac{\delta^3}{\mu}\alpha_4\beta_3\left[E_t(\pi_{t+3}) - \delta E_t(\pi_{t+4})\left(\frac{\beta_4}{\beta_3} - (1+\beta_1)\right)\right]\end{aligned}$$

| | | | | |
|---|---|---|---|---|
| Method: Generalized Method of Moments, Sample(adjusted): 1983:1 1997:3 | | | | |
| Bandwidth: Fixed (4) | | | | |
| Instrument list: $\pi_t$, $\pi_{t-1}$, $\pi_{t-2}$, $\pi_{t-3}$, $\pi_{t-4}$, $x_t$, $x_{t-1}$, $x_{t-2}$,$x_{t-3}$, $x_{t-4}$, | | | | |
| $\Delta_4 lpcm_t$, $\Delta_4 lpcm_{t-1}$, $\Delta_4 lpcm_{t-2}$, $\Delta_4 lpcm_{t-3}$, $\Delta_4 lpcm_{t-4}$, $i_t$, $i_{t-1}$ | | | | |
| | Coefficient | Std. Error | t-Statistic | Prob. |
| $c_0$ | 0.4032 | 0.1233 | 3.26 | 0.0018 |
| $\mu$ | 0.285 | 0.093 | 3.07 | 0.003 |
| | | | | |
| R-squared 0.737 | S.E. of regression 0.952 | | | |
| Mean dependent var 3.254 | J-statistic 0.17 | | | |

where $\delta$ has been fixed at .975.

# Figure 1




Actual and fitted real policy rates. The fitted rates are from the Euler equation under strict inflation targeting and real interest rate smoothing