

THE TREND AND THE MEASUREMENT OF WORLD INEQUALITY OVER EXTENDED PERIODS OF ACCOUNTING

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The principal result of this paper is the demonstration of a method for measuring an index of (e.g.) nominal GNP over extended periods of time and the inequality thereof. 'Snap shot' inequality for specific points in 1964, 1972 and 1980 is also reported and decomposed for six subgroups in 122 countries.

1. Introduction

The traditional practice of measuring inequality at specific points in time has long been subject to the criticism that such 'snap shots' reflect too many *transitory* effects. For a few countries attempts have been made to compute a measure of individuals' life or permanent incomes and the inequality in the distribution of such a measure. For example, while primarily concerned with 'mobility', Shorrocks (1980) computes U.S. inequality in the *total sum* of incomes over the period under study in the Michigan Panel Study of Income Dynamics. Here we compute four different aggregate measures of GNP (and per capita GNP) over the period 1964-1980. One of these measures corresponds roughly to the simple sum of GNP's, but the other three allow for different levels of *substitution* of GNP at different points in time. A further nuance here is the application of four different members of the Generalized Entropy (GE) class of decomposable inequality measures to international data. An extensive study considering several welfare attributes in addition to GNP is reported in Maasoumi and Jeong (1985).

The method of aggregation or averaging of incomes used here is based on the information theoretic approach of Maasoumi (1985). This technique is briefly discussed in the next section. The desirability and properties of the GE measures is discussed in the axiomatic treatment of Shorrocks (1984). This family includes both of Theil's measures and members which are ordinally equivalent to such other well known indices as coefficient of variation, Herfindahl, and the class of measures proposed by Atkinson (1970).

With developmental and geographical considerations in mind, the 122 countries in our sample were divided into six exclusive and exhaustive subgroups. Inequality was then decomposed into the between- and average within-group inequalities in order to present a more informative account of world inequality and its sources. Since the traditional approach is a special case of ours, we are able

to report inequalities for each of the constituent years (1964, 1972 and 1980), and to observe an increasing trend in inequality with between-group inequality being the dominant source in general.

2. Technical summary

Let X_{it} be the i th country's GNP at time t , $i = 1, \dots, N$, and $t = 1, \dots, T$. Let $X = (X_{it})$ and $x = (x_{it})$ denote the welfare and share matrices, respectively, where $x_{it} = p_i X_{it} / \sum_{i=1}^N p_i X_{it}$ and p_i is the population share of the i th country.¹ According to a generalized information measure of 'divergence', given in Maasoumi (1985), $S_t = h_t(x_{t1}, \dots, x_{tT})$ is an 'ideal' aggregate or average of the arguments of h_t , when

$$S_t = h \propto \left(\sum_{i=1}^T \alpha_i x_{it}^\beta \right)^{1/\beta}, \quad \beta \neq 0 \text{ or } -1, \quad (1)$$

with α_t being the evaluator's weights given to income at time t such that $\sum_t \alpha_t = 1$. The harmonic mean in eq. (1) is of course also a CES utility function with $\sigma = 1/(1 + \beta)$. $S_t \propto \prod_i x_{it}^{\alpha_i}$ and the linear $S_t \propto \sum_i \alpha_i x_{it}$ obtain, when generalized Kullback-Leibler measures ($\beta = 0$ and -1 , respectively) are the underlying information criteria.

The GE family of inequality measures may be defined as follows:

$$I_\gamma(S) = \sum_i p_i \left[(S_i^*/p_i)^{1+\gamma} - 1 \right] / \gamma(\gamma + 1), \quad \gamma \neq 0, -1, \quad (2)$$

$$\text{(Theil's second)} = \sum_i p_i \log p_i / S_i^*, \quad \gamma = -1,$$

$$\text{(Theil's first)} = \sum_i S_i^* \log S_i^* / p_i, \quad \gamma = 0,$$

where $S_i^* = p_i S_i / \sum_{i=1}^N p_i S_i$. The following type of decomposition is also available only for the GE family among the class of all measures that are scale invariant, symmetric and satisfy the Pigou-Dalton principle [see Shorrocks (1984)]. Let there be R groups, t_r , $r = 1, \dots, R$, containing N_r countries, and with population and attribute shares denoted by P_r and S_r , respectively. Then

$$I_\gamma(S) = I_\gamma(S.) + \sum_{r=1}^R P_r^{-\gamma} S_r^{1+\gamma} I_\gamma(S^r). \quad (3)$$

$I_\gamma(S.)$ denotes the between-group inequality, while the second term is a weighted sum (average, when $\gamma = 0$ or -1) of the inequalities within each subgroup ($I_\gamma(S^r)$). When $\gamma = -(1 + \beta)$, Propositions 1 and 2 of Maasoumi (1985) demonstrate that²

¹ Usually $p_i = 1/N$ is used. We report calculations using both $1/N$ and the actual population shares. We will distinguish the two sets of figures as, respectively, sample weighted (SW) and population weighted (PW) entries in our tables.

² The difference between the two sides of eq. (4) is a decreasing function of the minimum values of the information criteria that define S_t . As such, it may be considered as a measure of 'fit' of the aggregate distribution to those of the T constituent distributions. All the computations reported here are under this second decomposability condition ($\gamma = 1 - \beta$). Fuller detail is given in Maasoumi (1985).

$$I_\gamma(S) \leq \sum_{t=1}^T \mu_{t\gamma} I_\gamma(X_t) = \bar{I}, \quad \text{say.} \quad (4)$$

where $\sum_t \mu_{t\gamma} = 1$ only when $\gamma = 0$ or -1 . We conclude this section by noting that, (i) the most unambiguous decompositions in eqs. (3) and (4) are given by Theil's second measure ($\gamma = -1$), particularly when S_t is the Cobb-Douglas aggregator function, and (ii) the weights α_t are computed here as the normalized elements of the characteristic vector of $x'x$ corresponding to its largest characteristic root. See Ram (1982) for an application of the Principal Component (PC) method of obtaining 'composite indices' or aggregates of several attributes.³

3. International inequality

In our application we computed α as described above over the three points, $t = 1964, 1972$ and 1980 . S_t was computed as defined in eq. (1) for four values, $\sigma = \frac{1}{2}, 1$ (Cobb-Douglas), 2 and ∞ (linear). We computed the inequality in these four aggregates by four members of GE identified by, respectively, $\gamma = -2, 1$ (Theil's second), $-\frac{1}{2}$ and 0 (Theil's first). Sample weighted (SW) computations, with the PW figures (see footnote 1), are given in each table. Since population changes over time, the PW figures are based on the geometric mean of a country's population in 1964, 1972 and 1980. The latter figures as well as per capita GNP were taken from the World Bank (1966, 1974, 1982). The World Bank Atlas (WBA) method was used to obtain GNP figures in current U.S. dollars.⁴ Of the 125 countries with one or more millions in population, the data for three were not available in at least two of our chosen dates.⁵

Table 1 summarizes the group *shares* in population, GNP and the four aggregate GNP indices. By definition, when PW weights are used, we obtain GNP shares, and when SW weights are used we obtain the per capita GNP shares.

The following correlation matrices suggest that there is strong autocorrelation in the GNP series which, nevertheless, declines with time:

	Correlation - GNP per capita			Correlation - GNP		
	1964	1972	1980	1964	1972	1980
1964	1	0.985	0.892	1	0.994	0.945
1972		1	0.926		1	0.970
1980			1			1

Table 2 summarizes the inequality values (4) for each of the four GNP aggregates as well as inequality in each of the three constituent years. Table 3 gives the same results for *per capita GNP* shares (i.e., with $p_i = 1/122$). The first three rows of table 2 confirm the following observations:

³ Computing the vector $\alpha = (\alpha_1, \dots, \alpha_T)$ in this manner, it is the case that $\alpha'x = \sum_t \alpha_t x_{it}$ is the 1st PC of the x matrix, and thus accounts for the largest part of variation in x . Not surprisingly, the α_t 's are almost identical in our application, hence $\alpha'x$ amounts roughly to a simple average of GNP, while $\prod_t x_{it}^{\alpha_t}$ is close to its geometric mean.

⁴ Over the three years, weighted averages of domestic GNP deflators, implicit U.S. GNP deflator, and of exchange rates were used. The UN International Comparison Project (ICP) method was not used because of data problems. See Kravis et al. (1978) for a description of the ICP.

⁵ 1964 is the earliest data for which per capita GNP figures were available, and 1980 is the last date (before the conversion process changed) these figures were given by the World Bank for non-member countries.

Table 1
Share distribution of population and GNP per capita over time (total = 100).

	Population share			GNP share ^c			S_{γ}^{σ} ^a			
	1964	1972	1980	1964	1972	1980	($\sigma = \frac{1}{2}$)	($\sigma = 1$)	($\sigma = 2$)	($\sigma = \infty$)
<i>PW</i>										
Africa	7.2	7.8	8.4	1.7	1.7	2.1	1.7	1.8	1.8	1.8
Asia	26.3	27.5	28.1	4.7	3.6	4.0	4.0	4.1	4.1	4.1
Mid-east	4.9	5.4	5.6	2.3	2.7	5.0	2.9	3.1	3.2	3.4
L. America	6.9	7.5	7.8	4.4	5.0	5.7	5.1	5.1	5.1	5.0
Ind. market	19.1	18.1	16.4	64.21	67.4	64.9	65.7	65.6	65.6	65.5
Non-market	35.7	33.7	33.7	22.5	19.6	18.4	20.6	20.4	20.2	20.1
<i>PW^b</i>										
Africa		7.8		2.0	1.6	1.8	1.8	1.8	1.8	1.8
Asia		27.3		5.2	3.7	3.7	3.9	4.1	4.1	4.1
Mid-east		5.3		2.6	2.7	4.4	2.9	3.1	3.1	3.3
L. America		7.4		5.0	5.0	5.0	5.1	5.1	5.0	5.0
Ind. market		17.8		62.9	67.3	66.5	65.6	65.7	65.7	65.6
Non-market		34.4		22.2	19.8	18.6	20.6	20.4	20.3	20.1
<i>SW</i>										
Africa		27.0		6.4	5.6	4.1	5.2	5.3	5.3	5.3
Asia		12.3		4.0	4.0	4.2	4.0	4.0	4.0	4.1
Mid-east		14.8		13.7	13.7	18.4	14.0	14.6	15.0	15.3
L. America		17.2		12.6	11.3	9.1	11.0	11.0	11.0	10.9
Ind. market		15.6		48.6	52.7	53.3	53.0	52.4	52.0	51.6
Non-market		13.1		14.8	12.6	10.8	12.8	12.7	12.7	12.7

^a The PC weights are, respectively, 0.32, 0.33 and 0.35, i.e., roughly equal weights for GNP in the three years. Captured variation was 99.9%.

^b The PC weights used for this *weighted* geometric mean population were 0.34, 0.33 and 0.33. Captured variation was 94.9%.

^c The (mean, standard deviation) of per capita GNP for the three years are, respectively, (477.3, 604.6), (915.8, 1171.0) and (3036.0, 4120.0).

- (i) International inequality has increased over time according to every inequality measure (γ) used. Inequality in aggregate GNP ($I_{\gamma}(S)$) and for each year decreases with increasing values of σ and γ . This is to be expected as $-\gamma$ is the 'degree of constant relative inequality aversion' in the inequality measure. Thus Theil's first measure provides the lowest (but still high) inequality levels.
- (ii) Except for $I_{-2}(80)$, between-group inequality is always larger than the average within-group inequality, often by a factor of 2:1 or more. Between-group inequality decreases somewhat, while within-group inequalities decrease noticeably with increasing values of (σ, γ). (The biggest within-group inequality is that of the non-market countries which reflects the diversity of such nations in our grouping.) This further reveals the importance of the *between-group inequality as the primary source of total international inequality in GNP*.
- (iii) Concerning the aggregate measure, $I_{\gamma}(S)$, it conforms to every pattern noted in (i) and (ii) above, and is thus a very reliable summary measure. The upperbound values, \bar{I} , suggest an excellent fit of the S_{γ} series to the GNP over this interval which is clearly robust to changes to σ as well as γ . In this respect only $I_{\gamma}(72)$ is seen to perform as well, while $I_{\gamma}(64)$ or $I_{\gamma}(80)$ can give a misleading picture of inequality levels for this time interval. The advantage of $I_{\gamma}(S)$ is that it is not a 'snap shot' and hence can be used to evaluate inequality over extended periods of accounting.

Table 2
Generalized entropy international inequality in aggregated GNP and trends.

	$\sigma = 1, \gamma = -2$			$\sigma = 1, \gamma = -1$ (Theil's second)			$\sigma = 2, \gamma = -1$			$\sigma = \infty, \gamma = 0$ (Theil's first)					
	$I_\gamma(S)$	$I_\gamma(64)$	$I_\gamma(72)$	$I_\gamma(80)$	$I_\gamma(64)$	$I_\gamma(72)$	$I_\gamma(80)$	$I_\gamma(S)$	$I_\gamma(64)$	$I_\gamma(72)$	$I_\gamma(80)$	$I_\gamma(S)$	$I_\gamma(64)$	$I_\gamma(72)$	$I_\gamma(80)$
Total	1.872	1.369	1.909	2.538	0.941	0.825	0.969	1.086	0.811	0.749	0.838	0.789	0.759	0.826	0.847
Between-group	1.022	0.793	1.146	1.07	0.640	0.551	0.69	0.675	0.595	0.521	0.634	0.607	0.532	0.643	0.651
Within-group	0.85	0.576	0.762	1.469	0.301	0.273	0.276	0.411	0.216	0.228	0.204	0.182	0.227	0.183	0.196
\bar{I}		1.812				0.962				0.831				0.809	
<i>Developing</i> ^a															
Africa	0.276	0.248	0.286	0.435	0.26	0.254	0.286	0.335	0.270	0.275	0.306	0.293	0.313	0.344	0.325
Asia	0.086	0.027	0.087	0.214	0.096	0.032	0.104	0.233	0.108	0.036	0.121	0.264	0.129	0.041	0.323
Mid-east	0.170	0.102	0.232	0.434	0.176	0.116	0.218	0.347	0.198	0.133	0.233	0.350	0.241	0.165	0.383
L. America	0.100	0.126	0.105	0.119	0.08	0.113	0.090	0.082	0.076	0.111	0.087	0.073	0.075	0.112	0.067
Industrial	0.090	0.198	0.106	0.032	0.073	0.159	0.091	0.028	0.067	0.148	0.087	0.027	0.063	0.141	0.025
Non-market	0.981	0.757	0.746	1.389	0.658	0.570	0.564	0.852	0.602	0.536	0.532	0.762	0.588	0.529	0.741

^a 33 African countries, south of Sahara, 15 Asian, south east and Pacific, 18 in the Middle East including North Africa, Greece, Turkey and Portugal, 21 Latin American and Caribbean, 19 industrial members of OECD excluding south Europe, and 16 non-market countries of eastern Europe, China, Cuba, Vietnam, Kampuchea, Lao PDR and Mongolia.

Table 3
Generalized entropy international inequality in aggregated per capita GNP and trends.

	$\sigma = \frac{1}{2}, \gamma = -2$			$\sigma = 1, \gamma = -1$ (Theil's second)			$\sigma = 2, \gamma = -\frac{1}{2}$			$\sigma = \infty, \gamma = 0$ (Theil's first)						
	$I_{\gamma}(S)$	$I_{\gamma}(64)$	$I_{\gamma}(72)$	$I_{\gamma}(80)$	$I_{\gamma}(64)$	$I_{\gamma}(72)$	$I_{\gamma}(80)$	$I_{\gamma}(S)$	$I_{\gamma}(64)$	$I_{\gamma}(72)$	$I_{\gamma}(80)$	$I_{\gamma}(S)$	$I_{\gamma}(64)$	$I_{\gamma}(72)$	$I_{\gamma}(80)$	
Total	1.525	1.045	1.459	2.329	0.742	0.634	0.748	0.912	0.643	0.581	0.652	0.754	0.625	0.588	0.638	0.720
Between-group	0.696	0.543	0.646	0.888	0.472	0.401	0.463	0.549	0.439	0.380	0.439	0.499	0.439	0.384	0.448	0.495
Within-group	0.829	0.502	0.813	1.442	0.269	0.233	0.285	0.363	0.204	0.201	0.213	0.254	0.187	0.204	0.190	0.225
\bar{I}		1.548					0.768				0.668				0.650	
Developing ^a																
Africa	0.198	0.189	0.237	0.245	0.190	0.188	0.220	0.227	0.198	0.198	0.224	0.236	0.216	0.219	0.241	0.258
Asia	0.552	0.196	0.575	1.107	0.418	0.190	0.466	0.683	0.405	0.195	0.463	0.627	0.419	0.206	0.491	0.634
Mid-east	0.609	0.569	0.793	0.915	0.476	0.521	0.252	0.579	0.479	0.567	0.496	0.545	0.522	0.680	0.516	0.567
L. America	0.198	0.175	0.187	0.272	0.149	0.138	0.145	0.202	0.138	0.129	0.135	0.190	0.132	0.126	0.131	0.186
Industrial	0.061	0.095	0.069	0.053	0.052	0.080	0.060	0.046	0.048	0.075	0.058	0.044	0.047	0.073	0.056	0.042
Non-market	1.425	0.639	0.791	2.603	0.479	0.350	0.433	0.682	0.366	0.291	0.364	0.485	0.311	0.259	0.328	0.402

^a See table 2.

In table 3, observations (i) and (iii) hold without significant change. While average within-group inequalities are about the same as in table 2, the composition of within-group inequalities has changed somewhat, with greater inequality within the Asian and Mid-East countries. This is to be expected given that we are now dealing with *per capita* GNP shares. In one case ($\gamma = -2$) the average within group inequalities are even larger than the corresponding between-group terms. This can be seen to be essentially the consequence of a substantial increase in inequality *within* the non-market and Asian nations between 1964 and 1980. For any given year and, reassuringly, for the aggregate measure, inequalities decline with increasing values of γ , but there is relatively more stability in the between-group inequalities than in the within terms as γ increases to zero. $I_\gamma(72)$ again does well in mimicking $I_\gamma(S)$, while $I_\gamma(64)$ and $I_\gamma(80)$ do not. Finally, both between and within inequalities have increased over time.

4. Conclusion

We have provided an account of international inequality which is, in its broad outline, quite robust to the particular inequality measure chosen or the GNP aggregate adopted over the three dates. Our decompositions convey much more information than can be analysed in this space.

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