

Comparing Income Mobility in Germany and the US using Generalized Entropy Mobility Measures

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Abstract:

Based on a derivation of the asymptotic sampling distribution of the generalized entropy mobility measures, this paper provides a statistically rigorous analysis of income mobility in Germany and the United States using the panel data set PSID-SOEP equivalent data file. Several alternative measures of income aggregation, inequality measures, and groupings are considered to establish robustness. We find that, to a high degree of statistical confidence, post-government income mobility is much higher in Germany. Possible reasons for these findings are revealed through disaggregation of the samples by population subgroups.

Introduction

The interest in the dynamics of income distributions has grown with the increasing availability of panel data for many countries. It has been recognized that static snap-shots of income or earnings distributions alone are not sufficient for meaningful evaluation of welfare. A society with a rigid income distribution where everyone remains in the same position year after year is commonly regarded as less well-off than a “mobile” society. Therefore, mobility indices have developed and interpreted as indicators of “opportunity”.

There are many empirical studies of earnings and income mobility, see e.g. Björklund (1993), Burkhauser and Holtz-Eakin (1993), Burkhauser and Poupore (1997), Hungerford (1993), Gustafsson (1994), OECD (1996) and Schluter (1996). For an earlier survey of empirical studies of earnings mobility see Atkinson *et al.* (1992). Traditionally, empirical studies of earnings and income mobility have not addressed the issue of statistical significance and rigorous inference. Point “estimates” are reported but no confidence intervals. However, the commonly used panel data sets are rarely larger than, say, 15000

observations, over a short time frame, and therefore sampling errors should not be neglected a priori. We provide detailed formulae and the programming code necessary for statistical inference on a large class of mobility measures at a level which has come to dominate in other empirical areas.

There is a loosely related body of literature in econometrics which studies earnings/income “dispersion” employing standard panel data models. These studies are focused on explaining the sources of *variation* in earnings and should be interpreted with caution if “mobility” is to be inferred from them in a welfare-theoretic sense.

This paper reviews a large family of mobility measures, namely the generalized entropy mobility measures (GEMM) introduced by Maasoumi and Zandvakili (1986). The GEMM family is prominent in the tradition of “inequality reducing” class of mobility indices introduced by Shorrocks (1978). Shorrocks has demonstrated that members of GEMM that utilize the income aggregator functions in Maasoumi (1986) satisfy the greatest number of desirable properties for mobility measures. Also, recent deeper welfare-theoretic interpretations of the other tradition to mobility measurement, the transition matrix/Markov models, point to the same GEMM as an ideal family of indices in that context too. See Maasoumi (1998) for a synthesis.

We establish the asymptotic sampling distribution of these measures using the “delta method” and the existing theory of method of moments estimators. Since different individuals face different panel inclusion probabilities we also allow for weighted observations in our formulae. Similar results are available only for inequality measures, see Cowell (1989).

Our empirical application concerns the US and Germany before reunification. It may be viewed as a statistical investigation and broad generalization of the recent study of similar data by Burkhauser and Poupore (1997). These generalizations are: (i) They utilize two of the members in the class of measures studied in the present article (based on Theil entropies), we offer a robustification of findings over a broad class with different aversion parameters. (ii) To obtain “long run” incomes, they aggregate by adding up incomes over time, as was done in Shorrocks (1978). This assumes infinite substitutability of incomes at different points of the life cycle. This is a serious issue in the conception of “permanent income” and mobility. We follow Maasoumi and Zandvakili (1986) by considering a range of aggregator functions which allow for different degrees of substitution, including the simple sum. Again we study the sensitivity of our inferences to the aggregation method. (iii) We report standard errors which allow construction of confidence bands and tests of hypotheses regarding the levels and differences of mobility estimates, over time as well as between countries. Previous studies merely permit a numerical

comparison with quantities that may be casually judged as “too small” or “too large”. We find that some differences in the third decimal places are statistically significant.

Burkhauser and Poupore (1997) give an excellent review of the data, the reasons for interest in comparing the US and (West) Germany with their different labor markets and welfare schemes, and the type of mobility measures studied here. In particular they note that the existing econometric panel studies would be misread if the larger wage dispersion in the US is extrapolated to a finding of greater mobility in the US than Germany. Our findings offer general and strong *statistical* support for their conclusions that German income distributions exhibit greater degrees of mobility for the whole sample, as well as for almost all socio-economic sub-groups, defined by age, gender, and educational attainment. This is a statistical extension over a broader class of mobility measures, as well as more flexible “permanent income” functions.

The paper is structured as follows. Section 2 describes the generalized entropy mobility measures and derives their asymptotic distribution. Section 3 compares income mobility in Germany and the United States. Section 4 concludes.

Generalized Entropy Mobility Measures

The framework follows Maasoumi and Zandvakili (1986, 1990). Let Y_{it} denote income of person $i, i = 1, \dots, n$, in period $t = 1, \dots, T$, and $Y_t = (Y_{1t}, Y_{2t}, \dots, Y_{nt})'$. Let $S_i = S_i(Y_{i1}, \dots, Y_{iT})$ be the ‘permanent’ or aggregate income of individual i over T periods. This is just for convenience, of course, since one can define aggregate income over periods 1 to $T' \leq T$, say, and develop a mobility profile as T' approaches T . Then

$$S = (S_1, \dots, S_n)'$$

is the vector of aggregate incomes for a chosen time frame. Following Maasoumi (1986) the following type of aggregation functions are justified on the basis that they minimize the generalized entropy distance between S and all of the T income distributions:

$$\begin{aligned}
S_i &= \left(\sum_{t=1}^T \alpha_t Y_{it}^{-\beta} \right)^{-1/\beta} \quad \text{mbox } \beta \neq 0, -1 \\
&= \prod_{t=1}^T Y_{it}^{\alpha_t} \quad \text{mbox } \beta = 0 \\
&= \sum_{t=1}^T \alpha_t Y_{it} \quad \text{mbox } \beta = -1,
\end{aligned}$$

where α_t are the weights attached to income in period t , $\sum_t \alpha_t = 1$. The elasticity of substitution of income across time is constant at $\sigma = 1/(1 + \beta)$. The case $\beta = -1$ corresponds to perfect inter-temporal substitution which subsumes Shorrocks' analysis for certain weights. This case is also the most common formulation of the "permanent income" concept in economics and used by Burkhauser and Poupore (1997).

Mobility is measured as the ratio of "long run" income inequality occurring when the accounting period is extended, and a measure of short run inequality. The latter may be represented by any one period of interest, or a weighted average of the single period inequalities. Other than this notion of "equality enhancing" mobility, no other *welfare theoretic* bases have been put forward or implemented in favor of any other "mobility" or "dispersion" measures until recently footnote . The extension of the time interval is meant to reflect the dynamics and smooth out the transitory or life cycle effects. Shorrocks (1978) proposed the following mobility measures using the weighted short run inequalities, as follows:

$$M = 1 - \frac{I(S)}{\sum_{t=1}^T \alpha_t I(Y_t)}$$

where $I(\cdot)$ is the inequality measure. For convex inequality measures $I(\cdot)$, $0 \leq M \leq 1$ is easily verified when S is the linear permanent income function. For other aggregator functions see Maasoumi and Zandvakili (1990). A priori, there would be no reason for an analyst to give unequal weights to different years under study. Nevertheless, Shorrocks (1978) suggests the ratio of year t income to total income over the T periods as suitable values for α_t s. We consider both weighting schemes here footnote .

The family of inequality measures used here is the generalized entropy (GE). For a weighted random vector $X = (X_1, \dots, X_n)'$ with weights $w = (w_1, \dots, w_n)'$ the generalized entropy inequality measure is defined as

$$I_\lambda(X) = \frac{1}{\lambda(\lambda+1)} \frac{1}{n} \sum_{i=1}^n \left(\frac{w_i}{\bar{w}} \right) \left[\left(\frac{X_i}{\bar{X}} \right)^{\lambda+1} - 1 \right] \quad \text{for } \lambda \neq 0, -1$$

where $\bar{X} = \frac{1}{n} \sum (w_i/\bar{w})X_i$ and $\bar{w} = \sum w_i/n$. Usually (as in our application) the weights are the reciprocal inclusion probabilities. For $\lambda = 0, -1$ this index converges to the first and second Theil measures of information, respectively:

$$I_0(X) = \frac{1}{n} \sum_{i=1}^n \left(\frac{w_i}{\bar{w}} \right) \left(\frac{X_i}{\bar{X}} \right) \ln \left(\frac{X_i}{\bar{X}} \right)$$

$$I_{-1}(X) = -\frac{1}{n} \sum_{i=1}^n \left(\frac{w_i}{\bar{w}} \right) \ln \left(\frac{X_i}{\bar{X}} \right).$$

The asymptotic distribution of GEMM:

Cowell (1989) noted that the GE measures of inequality are functions of the moments of the income variable. Similarly, we note that the GE mobility measures can be written as functions of moments of (transformed) income variables. Let $\mu(X) = \int_0^\infty x dF(x)$, for any variable X . Then, for $\lambda \neq 0, -1$

$$M_\lambda = M_\lambda(\mu(Y_1), \dots, \mu(Y_T), \mu(S), \mu(Y_1^{\lambda+1}), \dots, \mu(Y_T^{\lambda+1}), \mu(S^{\lambda+1}))$$

$$= 1 - \frac{\frac{\mu(S^{\lambda+1})}{\mu^{\lambda+1}(S)} - 1}{\sum_{t=1}^T \alpha_t \left(\frac{\mu(Y_t^{\lambda+1})}{\mu^{\lambda+1}(Y_t)} - 1 \right)}$$

and

$$M_0 = M_0(\mu(Y_1), \dots, \mu(Y_T), \mu(S), \mu(Y_1 \ln(Y_1)), \dots, \mu(Y_T \ln(Y_T)), \mu(S \ln(S)))$$

$$= 1 - \frac{\frac{\mu(S \ln(S))}{\mu(S)} - \ln(\mu(S))}{\sum_{t=1}^T \alpha_t \left(\frac{\mu(Y_t \ln(Y_t))}{\mu(Y_t)} - \ln(\mu(Y_t)) \right)}$$

$$M_{-1} = M_{-1}(\mu(Y_1), \dots, \mu(Y_T), \mu(S), \mu(\ln(Y_1)), \dots, \mu(\ln(Y_T)), \mu(\ln(S)))$$

$$= 1 - \frac{\ln(\mu(S)) - \mu(\ln(S))}{\sum_{t=1}^T \alpha_t (\ln(\mu(Y_t)) - \mu(\ln(Y_t)))}$$

Let $\boldsymbol{\mu}$ be a vector of moments and $M = M(\boldsymbol{\mu})$ a differentiable function. Then, for general processes the following asymptotic distribution is well known for $\hat{\boldsymbol{\mu}}$, a (weighted) estimator of $\boldsymbol{\mu}$, (e.g., see Amemiya's (1985) textbook) :

$$\sqrt{\frac{n}{1 + v_w^2}} (\hat{\boldsymbol{\mu}} - \boldsymbol{\mu}) \xrightarrow{d} N(0, \Sigma)$$

with $\Sigma = \text{Cov}(\hat{\boldsymbol{\mu}})$ and v_w^2 the coefficient of variation of the weights w . By an application of the Delta method it follows that:

$$\sqrt{n} \frac{M(\hat{\boldsymbol{\mu}}) - M(\boldsymbol{\mu})}{(1 + v_w^2)^{\frac{1}{2}} \sigma_M} \xrightarrow{d} N(0, 1)$$

where

$$\sigma_M^2 = \left(\frac{\partial M}{\partial \boldsymbol{\mu}} \right)' \Sigma \left(\frac{\partial M}{\partial \boldsymbol{\mu}} \right)$$

with $\left(\frac{\partial M}{\partial \boldsymbol{\mu}} \right)$ denoting the derivative vector. Of course, Σ is usually unknown and has to be substituted by a suitable estimator $\hat{\Sigma}$; also, the gradient will have to be evaluated at $\hat{\boldsymbol{\mu}}$. However, this does not alter the asymptotic distribution. See, for example, Amemiya (1985).

The partial derivatives and the asymptotic covariance matrices mentioned above are derived in the Appendix to this paper. A GAUSS program implements the computations.

Empirical Application

This section is concerned with income mobility in Germany and the United States in the second half of the 1980s. The reunification of Germany after 1989 is apt to produce income distributions which are transitory. Mobility measures employed here are designed to avoid temporal effects. The study of the post reunification data, as a new regime, deserves a separate study. Our data are taken from the PSID and the GSOEP equivalent data files. These files provide data about many characteristics of individuals between 1984 and 1989. In particular there are comparable pre- and post-government income variables for both countries (see Butrica and Jurkat (1998) for detailed descriptions of the data sets). Our data are essentially the same as the ones described and analyzed in Burkhauser and Poupore (1997). All of our results are obtained for the individual level figures, using equivalence scales when necessary. Burkhauser and Poupore report point estimates of Gini and two members of our mobility family, and they use only a simple sum of incomes over time for their long run income measure. It is known that Gini, as well as some GEMM indices (with aversion parameter λ close to that of Gini) are relatively insensitive to the tail areas of distributions. This makes them unrevealing when policy programs are

often directed toward these tails (especially the lower). Our application covers the cases considered by Burkhauser and Poupore, but includes a sensitivity analysis by considering other values for the “inequality aversion” parameter, as well as the income aggregation functions. We also solidify the empirical findings by statistical tests of significance, as well as provide a new dynamic mobility graph.

Data

Pre-government income is the sum of total family income from labor earnings, asset flows, private transfers, and the imputed rental value of owner occupied housing. Labor earnings include wages and salary from all employment including training, self-employment income, and bonuses, overtime, professional practice or trade, and profit-sharing. Asset flows include income from interest, dividends, and rent. In Germany private transfers include payments from individuals outside of the household. These include alimony and child support payments. In the US private pensions, Veterans Administration pensions, and annuities are also included. Estimation of the imputed value of housing differs between both countries: In Germany imputed rental value of owner occupied housing is a respondent supplied estimate of the monthly rental value of their dwelling. This estimate is annualized. In the US imputed rental value of owner occupied housing is 6.0 percent of the difference between the house value and the remaining mortgage principle.

Post-government income is the sum of total family income from labor earnings, asset flows, private transfers, public transfers, and the imputed rental value of owner occupied housing, minus total family taxes. Earnings, asset flows, private transfers and imputed rental value of housing are defined as in pre-government income. In Germany public transfers include housing allowances, child benefits, subsistence allowance from the Social Welfare Authority, special circumstances benefits from the Social Welfare Authority, Social Security pensions for old age, disability, or widowhood, government student assistance, maternity benefits, unemployment benefits, unemployment assistance, and unemployment subsistence allowance. In the US public transfers include AFDC payments, supplemental security income (SSI), social security payments, unemployment compensation, worker’s compensation, and the face value of food stamps.

For Germany, total family taxes are derived by micro simulation routines developed by Schwarze (1995). They include income taxes, church taxes, and social security payments (health, unemployment, and retirement insurances). These routines assume that, all married couples file jointly, all filing units take

the standard deductions and no other deductions, and all employees are subject to the average national contribution rates for health, unemployment, and retirement insurance. Under these assumptions, rather accurate estimates of the tax burden are obtained for Germany. Post-government (post.gov) income data are obtained using these estimates.

For the US, total family taxes includes income taxes of the head, partner and other family members, as well as payroll taxes of the head and partner. Income tax values are provided by the PSID. Payroll taxes are calculated by bracketing labor income and applying the average payroll tax rate for that bracket as reported by the Social Security Bulletin, Annual Statistical Supplement (1990).

Other variables which we used are the number of persons in the household, age, years of education, participation level in the labor market, and sex of the individual. The data set offers two equivalence scales, the OECD scale and the ELES scale.

Inspecting the data closely, one finds that some observations are implausible. For instance, it is not likely to survive on an annual post-government income of merely 1000 DM. Some GE inequality and mobility measures are known to be especially non-robust to measurement errors in the tails of income distributions, see e.g., Cowell and Victoria-Feser (1996), and Cowell and Schluter (1998). In order to avoid this kind of sensitivity we excluded individuals with annual *post-government equivalent* incomes of less than US\$450 and DM 1000. The proportion of deletions is always less than 1.1 percent of the number of observation.

Inequality Results

As a first step it is necessary and informative to analyze the GE indices of inequality for both countries. We have exemplified these results in table 1. We report aggregator functions and inequality measures which are different from the ones reported in Burkhauser and Poupore (1997). This is merely to economize on space. Our results confirm their findings, and more extensive results are available from the authors which demonstrate a qualitative robustness to the choice of inequality measure and the aggregation functions.

For the PSID, the left hand panels of table 1 report GE *inequality of nominal post government* income equivalized by the OECD scale. Two representative parameter combinations are considered to reflect different degrees of inequality aversion/inter-temporal substitution. Inequality is reported for each single year and for aggregated income over increasing number of years. The row “from” indicates the starting period, and the column “until” reveals the last year of aggregation. As an example, with $\lambda = 0$ and $\beta = -1$ (Theil’s measure over

simple sum of incomes), the inequality of incomes aggregated over 1986-1988 is 0.2327. Standard errors are in parentheses.

For the SOEP, the right hand panels of table 1 depict the GE inequality measures for the same income variable as above. Consistent with the PSID entries, the changes of the inequality values are generally statistically significant. Given the nature of approximations in the Delta method, the high degree of significance in most of these entries is comforting. In particular we note that income inequality in the US first declines, then increases significantly in the latter years of the 80s. So much so, that aggregated income inequality increases when these latter years are included.

[B] tabcolsep 0.3mm caption

from	PSID						SOEP				
	$\beta = -1$ and $\lambda = 0$										
	until						until				
	1984	1985	1986	1987	1988	1989	1984	1985	1986	1987	1988
1984	.2086 (.0054)	.2079 (.0075)	.2061 (.0076)	.2040 (.0074)	.2106 (.0080)	.2223 (.0093)	.1308 (.0040)	.1126 (.0033)	.1052 (.0033)	.0983 (.0028)	.0941 (.0026)
1985		.2380 (.0101)	.2230 (.0088)	.2158 (.0080)	.2225 (.0085)	.2335 (.0099)		.1279 (.0047)	.1162 (.0046)	.1052 (.0034)	.0998 (.0030)
1986			.2371 (.0073)	.2236 (.0068)	.2327 (.0083)	.2443 (.0101)			.1333 (.0089)	.1118 (.0041)	.1051 (.0031)
1987				.2417 (.0068)	.2502 (.0092)	.2595 (.0112)				.1130 (.0028)	.1084 (.0029)
1988					.2910 (.0135)	.2860 (.0138)					.1231 (.0037)
1989						.3056 (.0137)					
from	$\beta = 0$ and $\lambda = -1$										
	until						until				
		1984	1985	1986	1987	1988	1989	1984	1985	1986	1987
1984	.2198 (.0041)	.2086 (.0042)	.2062 (.0043)	.2030 (.0043)	.2014 (.0044)	.2069 (.0047)	.1259 (.0028)	.1067 (.0022)	.0986 (.0019)	.0935 (.0019)	.0915 (.0019)
1985		.2326 (.0053)	.2211 (.0049)	.2138 (.0045)	.2114 (.0046)	.2164 (.0050)		.1230 (.0031)	.1086 (.0024)	.1003 (.0021)	.0974 (.0021)
1986			.2415 (.0048)	.2259 (.0044)	.2226 (.0046)	.2272 (.0051)			.1232 (.0044)	.1056 (.0022)	.1023 (.0022)
1987				.2455 (.0047)	.2360 (.0049)	.2385 (.0055)				.1166 (.0025)	.1105 (.0024)
1988					.2651 (.0067)	.2574 (.0067)					.1242 (.0029)
1989						.2760 (.0069)					

[E]

We also find that inequality is statistically significantly lower in Germany than the US, at any level of aggregation. These findings are predictive of the mobility profiles.

Mobility results

Looking at overall post-government mobility in both countries we find that income mobility is clearly higher numerically in Germany than in the US. These differences are statistically significant at all reasonable levels. We report only a few typical tables here, but *this finding is robust with respect to the weighting scheme, the consideration of inflation, the equivalence scale, the aggregation method (β) and the inequality measure (λ)*. These provide significant and extensive generalizations of the Burkhauser and Poupore's numerical results.

Both short-run and long-run mobility are higher in Germany. It seems that aggregating income over six years or so gives a reliable picture of lifetime inequality in the US. In Germany the aggregation period would have to be longer — how much longer cannot be inferred from the available sample period.

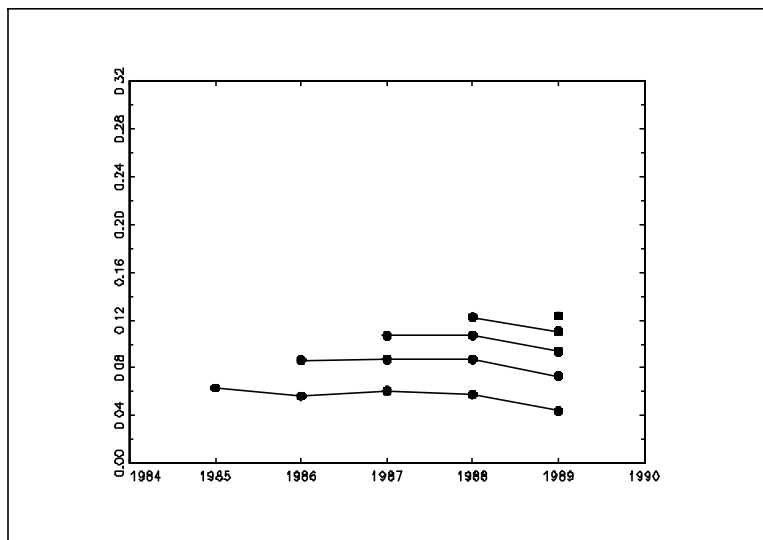
Table 2 shows the generalized entropy *mobility* measures for the two countries. For instance, taking $\lambda = 0, \beta = -1$ as in Burkhauser and Poupore (1997), over the period 1985-1988, mobility estimates for Germany and the US are 0.1809 and 0.1071, respectively. Standard errors based on the formulae derived in the previous section are reported in parentheses. The asymptotic distribution theory suggests that these figures are statistically significant. A relatively plausible assumption of independence between these two samples allows for quick computations and the statistical rejection of the null hypothesis of equality between these two estimates. In this example inflation was not taken into account and the equivalence scale is the OECD scale.

[B] caption

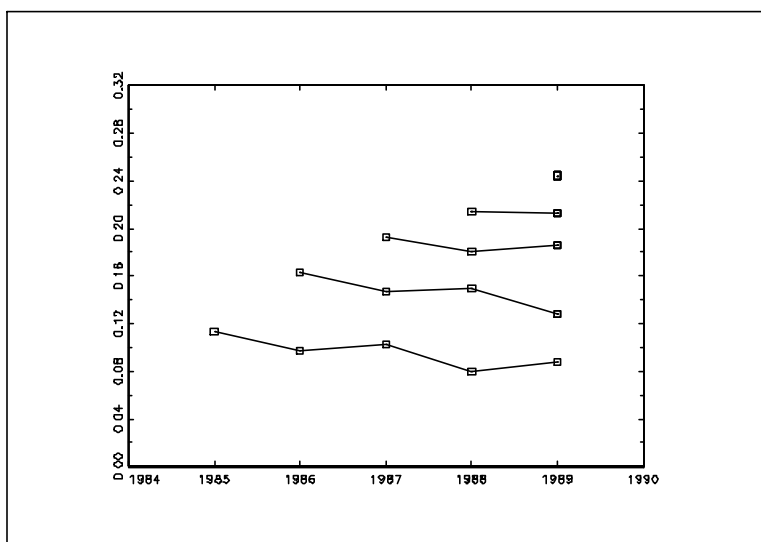
from	PSID					SOEP				
	$\beta = -1$ and $\lambda = 0$									
	until					until				
	1985	1986	1987	1988	1989	1985	1986	1987	1988	1989
1984	.0630 (.0030)	.0865 (.0034)	.1067 (.0040)	.1223 (.0041)	.1235 (.0048)	.1137 (.0045)	.1629 (.0057)	.1925 (.0069)	.2142 (.0079)	.2443 (.0082)
1985		.0563 (.0024)	.0870 (.0030)	.1071 (.0035)	.1106 (.0042)		.0975 (.0054)	.1467 (.0076)	.1809 (.0087)	.2127 (.0092)
1986			.0607 (.0024)	.0871 (.0030)	.0935 (.0039)			.1027 (.0079)	.1499 (.0096)	.1854 (.0102)
1987				.0575 (.0022)	.0731 (.0034)				.0801 (.0035)	.1278 (.0046)
1988					.0439 (.0027)					.0878 (.0045)
from	$\beta = 0$ and $\lambda = -1$									
	until					until				
		1985	1986	1987	1988	1989	1985	1986	1987	1988
1984	.0679 (.0029)	.0933 (.0029)	.1147 (.0031)	.1386 (.0038)	.1495 (.0040)	.1231 (.0053)	.1737 (.0075)	.1984 (.0078)	.2158 (.0076)	.2463 (.0077)
1985		.0583 (.0019)	.0918 (.0025)	.1196 (.0034)	.1313 (.0035)		.1088 (.0086)	.1536 (.0092)	.1825 (.0085)	.2154 (.0082)
1986			.0619 (.0021)	.0969 (.0029)	.1108 (.0029)			.1142 (.0121)	.1525 (.0101)	.1889 (.0092)
1987				.0668 (.0028)	.0877 (.0025)				.0816 (.0036)	.1355 (.0050)
1988					.0510 (.0023)					.0909 (.0049)

[E]

Figures 1 and 2 depict the same indices (with $\lambda = 0$ and $\beta = -1$). The lowest lines show the two-period mobility, the next lines show the three-period mobility, etc. Note that the *traditional* mobility profile – i.e., extending the accounting period while keeping the starting period fixed, is obtained by connecting the first points of each line.



GE mobility index for the PSID
unrecognized



We agree with Burkhauser and Poupore that these findings demonstrate that the inferences made about earnings/wage dispersion in the panel data studies fail to reflect “mobility” that is produced in Germany by government programs and taxes. The explanation is that our measures of mobility are conceived in terms of the usual increasing and Schur-concave (equality preferring) Social Welfare Functions which acknowledge a subjective direction to income movements. Variance and dispersion are questionable measures of mobility. They are merely indicators that suggest the potential for higher mobility in the less rigid and less

organized labor markets of the US. This potential is not realized.

Our comparative ranking inferences are not particular to group characteristics. In order to explore this question in our approach, one decomposes mobility measures. The parallel to this is the inclusion of the corresponding characteristics as covariates in regression studies. Thus, we split the samples according to various criteria and investigate mobility within each group (we did not take into account income mobility between groups). We note that the ordering is robust relative to adjustment for inflation by the consumer price indices.

It turns out that many group characteristics do not have a significant impact on comparative income mobility profiles. For instance, looking at males and females separately, the mobility pictures look very similar to the ones in figures 1 and 2. The same may be said of which equivalence scale is used, OECD or ELES. Using household income instead of equivalized individual income, we find that the mobility estimates are somewhat lower for Germany but unchanged for the US. The same phenomenon was observed using per capita income.

Age

To investigate the impact of age, we split the sample into six age groups. Table 3 clarifies the age classes and their sample shares averaged over the 1984-1989 period. The age structure of the two countries differ.

Age class	PSID	SOEP
0 – 15	0.232	0.154
16 – 25	0.122	0.131
26 – 35	0.160	0.111
36 – 50	0.116	0.169
51 – 65	0.074	0.112
66 +	0.050	0.056

Tables 4 and 5 show the GEMM for each of the age groups for both countries. The parameter combinations correspond to Theil's first measure of inequality, and the arithmetic income aggregators. This facilitates direct comparison with the Burkhauser-Poupore findings. Other values have been tried and are available from us.

[B] caption

[E]

[B] caption

	PSID					SOEP				
	until					until				
Age 36–50										
1984	.0628 (.0048)	.0918 (.0063)	.1143 (.0067)	.1266 (.0088)	.1180 (.0122)	.1041 (.0089)	.1380 (.0101)	.1657 (.0115)	.1693 (.0111)	.1956 (.0112)
1985		.0705 (.0056)	.0999 (.0062)	.1154 (.0085)	.1078 (.0116)		.0790 (.0088)	.1227 (.0112)	.1455 (.0106)	.1677 (.0103)
1986			.0736 (.0061)	.0980 (.0079)	.0994 (.0104)			.0700 (.0065)	.1194 (.0093)	.1405 (.0088)
1987				.0538 (.0046)	.0695 (.0080)			.0779 (.0067)	.1052 (.0070)	
1988					.0417 (.0064)					.0606 (.0065)
Age 51–65										
1984	.0447 (.0084)	.0604 (.0098)	.0754 (.0117)	.0993 (.0145)	.1002 (.0157)	.0832 (.0068)	.1412 (.0152)	.1645 (.0199)	.1854 (.0247)	.2189 (.0247)
1985		.0393 (.0059)	.0639 (.0092)	.0932 (.0130)	.0972 (.0147)		.1024 (.0160)	.1433 (.0226)	.1741 (.0275)	.2128 (.0270)
1986			.0463 (.0068)	.0814 (.0102)	.0849 (.0120)			.1098 (.0220)	.1600 (.0294)	.1968 (.0294)
1987				.0620 (.0083)	.0708 (.0109)			.0807 (.0080)	.1342 (.0095)	
1988					.0420 (.0084)					.0844 (.0088)
Age 66+										
1984	.0718 (.0114)	.0740 (.0062)	.0896 (.0079)	.0927 (.0067)	.0948 (.0099)	.1474 (.0179)	.1671 (.0202)	.1778 (.0255)	.1745 (.0210)	.1911 (.0232)
1985		.0335 (.0073)	.0657 (.0050)	.0691 (.0070)	.0733 (.0103)		.0584 (.0093)	.1133 (.0210)	.1180 (.0147)	.1291 (.0167)
1986			.0488 (.0061)	.0616 (.0050)	.0664 (.0078)			.0794 (.0273)	.0814 (.0140)	.1071 (.0127)
1987				.0581 (.0079)	.0637 (.0057)			.0567 (.0107)	.0940 (.0113)	
1988					.0418 (.0064)					.0807 (.0127)

[E]

As can be gleaned from these results, age is a rather important source of heterogeneity in mobility indicators. For both countries, mobility is generally highest for the second age group (young adults aged 16-25) and decreasing thereafter. This is consistent with prior expectations regarding life cycle earning patterns, and one of the main reasons one prefers to look at mobility rather than static single year estimates of inequality. For all age groups, mobility is statistically significantly higher in Germany than in the US, sometimes dramatically so. Trede (1998) obtains similar findings for the “earnings” profiles in West Germany. This phenomenon was noted by Burkhauser and Poupore (1997), and is contrary to somewhat misplaced prior expectations that freer US labor markets will lead to greater mobility compared to more rigid markets in Germany, emphasizing credentials and union-management negotiated wage levels. Our inferences put these contrary findings on a statistically sound and comparable basis with those gleaned from the existing econometric studies which often find statistically larger dispersions for the US earnings.

Conclusion

We have derived the asymptotic distribution of the GE measures of mobility proposed by Shorrocks-Maasoumi-Zandvakili. Our results cover the case of independent but not necessarily identical processes. Our results also allow for the common phenomenon of weighted observations, for instance, by inclusion probabilities.

Our primary focus was the post-government income mobility in Germany and the US. Our main empirical finding is that mobility estimates are statistically significantly higher in Germany than the US. We find that this ordering exists for all of our six age groups. Our preliminary look at pre-government income distributions has so far indicated the same orderings.

We have also looked at the traditional mobility profiles and a “moving” profile. The traditional graphical analysis is supplemented and supported by our statistical techniques and findings. We find that, for the US, the estimated profile becomes almost flat after six years. This may suggest that a six year aggregation period is adequate for representing “long run” incomes in the US. In contrast, the German mobility profile is steeper and has a positive slope even at the end of the six years. Longer sample periods are required to study this issue further. But our analysis stops at about the German reunification. The consequent structural changes deserve a separate study after sufficient time has elapsed.

One welfare implication of these results is that the combination of higher annual inequality and lower mobility in the US indicates a higher ranking for

Germany in these respects. But given that our inequality measures are *relative indices*, these inferences neglect the higher average income in the US and should not be used to indicate welfare *levels* in the two countries. This is best done using tests of stochastic dominance and Generalized Lorenz relations. See Maasoumi and Heshmati (2000).

Appendix

Lemma 1. The partial derivatives for M_λ , $\lambda \neq 0, -1$ are:

$$\begin{aligned} \frac{\partial M_\lambda}{\partial \mu(Y_t)} &= -\frac{\alpha_t(\lambda + 1) \left(\frac{\mu(S^{\lambda+1})}{\mu^{\lambda+1}(S)} - 1 \right) \mu(Y_t^{\lambda+1})}{\left(\sum_{t=1}^T \alpha_t \left(\frac{\mu(Y_t^{\lambda+1})}{\mu^{\lambda+1}(Y_t)} - 1 \right) \right)^2 \mu^{\lambda+2}(Y_t)} \\ \frac{\partial M_\lambda}{\partial \mu(S)} &= \frac{(\lambda + 1) \mu(S^{\lambda+1})}{\mu^{\lambda+2}(S) \sum_{t=1}^T \alpha_t \left(\frac{\mu(Y_t^{\lambda+1})}{\mu^{\lambda+1}(Y_t)} - 1 \right)} \\ \frac{\partial M_\lambda}{\partial \mu(Y_t^{\lambda+1})} &= \frac{\alpha_t \left(\frac{\mu(S^{\lambda+1})}{\mu^{\lambda+1}(S)} - 1 \right)}{\left(\sum_{t=1}^T \alpha_t \left(\frac{\mu(Y_t^{\lambda+1})}{\mu^{\lambda+1}(Y_t)} - 1 \right) \right)^2 \mu^{\lambda+1}(Y_t)} \\ \frac{\partial M_\lambda}{\partial \mu(S^{\lambda+1})} &= -\frac{1}{\mu^{\lambda+1}(S) \sum_{t=1}^T \alpha_t \left(\frac{\mu(Y_t^{\lambda+1})}{\mu^{\lambda+1}(Y_t)} - 1 \right)} \end{aligned}$$

For M_0 the derivatives are

$$\begin{aligned} \frac{\partial M_0}{\partial \mu(Y_t)} &= -\frac{\alpha_t \left(\frac{\mu(S \ln(S))}{\mu(S)} - \ln(\mu(S)) \right) \left(\frac{\mu(Y_t \ln(Y_t))}{\mu(Y_t)} + 1 \right)}{\mu(Y_t) \left(\sum_{t=1}^T \alpha_t \left[\frac{\mu(Y_t \ln(Y_t))}{\mu(Y_t)} - \ln(\mu(Y_t)) \right] \right)^2} \\ \frac{\partial M_0}{\partial \mu(S)} &= \frac{\frac{\mu(S \ln(S))}{\mu(S)} + 1}{\mu(S) \sum_{t=1}^T \alpha_t \left[\frac{\mu(Y_t \ln(Y_t))}{\mu(Y_t)} - \ln(\mu(Y_t)) \right]} \\ \frac{\partial M_0}{\partial \mu(Y_t \ln(Y_t))} &= \frac{\alpha_t \left(\frac{\mu(S \ln(S))}{\mu(S)} - \ln(\mu(S)) \right)}{\mu(Y_t) \left(\sum_{t=1}^T \alpha_t \left[\frac{\mu(Y_t \ln(Y_t))}{\mu(Y_t)} - \ln(\mu(Y_t)) \right] \right)^2} \\ \frac{\partial M_0}{\partial \mu(S \ln(S))} &= -\frac{1}{\mu(S) \sum_{t=1}^T \alpha_t \left[\frac{\mu(Y_t \ln(Y_t))}{\mu(Y_t)} - \ln(\mu(Y_t)) \right]}, \end{aligned}$$

and for M_{-1} ,

$$\begin{aligned}\frac{\partial M_{-1}}{\partial \mu(Y_t)} &= \frac{\alpha_t(\ln(\mu(S)) - \mu(\ln(S)))}{\mu(Y_t) \left(\sum_{t=1}^T \alpha_t [\ln(\mu(Y_t)) - \mu(\ln(Y_t))] \right)^2} \\ \frac{\partial M_{-1}}{\partial \mu(S)} &= -\frac{1}{\mu(S) \sum_{t=1}^T \alpha_t [\ln(\mu(Y_t)) - \mu(\ln(Y_t))]} \\ \frac{\partial M_{-1}}{\partial \mu(\ln(Y_t))} &= -\frac{\alpha_t(\ln(\mu(S)) - \mu(\ln(S)))}{\left(\sum_{t=1}^T \alpha_t [\ln(\mu(Y_t)) - \mu(\ln(Y_t))] \right)^2} \\ \frac{\partial M_{-1}}{\partial \mu(\ln(S))} &= \frac{1}{\sum_{t=1}^T \alpha_t [\ln(\mu(Y_t)) - \mu(\ln(Y_t))]}.\end{aligned}$$

Lemma 2. The covariance matrix Σ can be found using the formula:

$$\text{mbox}(\hat{\mu}(X_1), \hat{\mu}(X_2)) = (1 + v_w^2) \frac{\text{mbox}(X_1, X_2)}{n}$$

where X_1, X_2 are to be replaced by the variables $Y_t, Y_t^{\lambda+1}, Y_t \ln(Y_t)$, or $\ln(Y_t)$ and $S, S^{\lambda+1}, S \ln(S)$, or $\ln(S)$, as appropriate.

These formulae have been implemented in GAUSS.

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