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A COMPARISON OF GRF AND OTHER REDUCED-FORM ESTIMATORS IN SIMULTANEOUS EQUATIONS MODELS*

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Numerical evidence is reported here in order to shed light on the characteristics and performance of several non-traditional reduced-form estimators. Some empirical and Monte Carlo simulation examples compare these techniques with such traditional methods as the OLS, 2SLS, 3SLS and FIML. While further evidence is required to more fully characterize the new estimators and the situations in which they can be successfully applied, the available evidence favors the new methods over the traditional ones. This is particularly so for the Generic and the Modified Stein-like Reduced Form (GRF and MSRF) estimators which, under our criteria, out-perform the traditional methods even in situations that are favorable to the latter.

1. Introduction

In this paper we provide some numerical evidence to help characterize the performance and the statistical properties of several reduced-form estimators. The new estimators investigated here are the Partially Restricted Reduced-Form (PRRF), the Modified Stein-like Reduced-Form (MSRF) and the Generic Reduced-Form (GRF) estimators. This focus on reduced-form estimators reflects our belief that they are key to policy simulations and forecast reliability. The quality of model formulation is, however, another key ingredient that must be taken into account.

It is widely understood that some of the information used for model specification is not amenable to statistical testing and inference. Different degrees of belief in the maintained model thus lead to alternative statistical techniques and approaches. For instance, in the Simultaneous Equations Models (SEMs), standard estimation techniques may be appropriate if all of the incorporated non-sample information is believed with certainty. If there is some uncertainty, however, alternative approaches are needed which differ only by the degree and the manner in which 'non-sample' information is

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incorporated in them. The implementation of the ideal of purely data-based, profligate statistical modelling and inference, exemplified by the recent example of vector autoregressive models (VARs) [see Sims (1980)], is severely curtailed by practical data limitations and other statistical difficulties. These difficulties lead inevitably to the assumption of different but equally incredible a priori restrictions that are no more amenable to statistical testing than those routinely incorporated in the traditional SEM methods.

There can be little doubt that more robust statistical methods are needed that do not entirely depend on the complete accuracy of non-sample information. Convenient formalisms that can be helpful in modelling such inherent uncertainties are essentially Bayesian or have attractive Bayesian interpretations. In the context of SEMs there are now several non-traditional methods which have particular Bayesian interpretations that are consistent with the natural evolution of views and of a priori information in economic modelling.[1]

The theoretical properties of these methods have been studied in recent years. Byron (1976), Knight (1977) and Swamy and Mehta (1980) have studied the Partially Restricted Reduced-Form (PRRF) estimator of Amemiya–Court–Kakwani. Maasoumi (1978) proposed and analyzed the MSRF estimator, and Maasoumi (1986) proposed and analyzed the GRF model and estimation methods. In this paper we report the available evidence on the performance of these techniques and compare them with the Unrestricted Least Squares (ULS) and the *derived* OLS, 2SLS, 3SLS and FIML reduced-form estimators. This evidence is based both on empirical models which may be regarded as inadequate or uncertain, and on Monte Carlo experiments on a limited number of both correctly and incorrectly specified models.

We find that a whole range of criteria favor the GRF and MSRF estimators. Subject to the inherent specificity of all numerical evidence we are prepared to give a qualified recommendation of GRF and MSRF over all the traditional methods.

Section 2 describes the classical SEM and its standard estimators. Section 3 briefly describes the non-traditional methods. Summary of the available evidence appears in section 4, and section 5 concludes.

2. The classical SE model

Let $X=[Y\ Z]$ represent T observations on n endogenous (Y) and m exogenous (Z) variables. Consider the following relationships:

$$Y' = PZ' + V', \tag{1}$$

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[1] In the case of macrovariables the 'Bayesian' extensions of the VAR models appear to be in violation of the elements of Bayesian analysis. The same sample of macrovariables (e.g., for the U.S. economy) may not be allowed to provide both the 'a priori' or subjective beliefs and the current sample information! Data-based priors must depend on samples other than the objective likelihood.

and

$$BY' + \Gamma Z' = U' = AX', \quad \text{say}, \tag{2}$$

where $A = (B \;\; \Gamma)$ being $n \times (n+m)$ and P being $n \times m$ represent the unknown coefficients. Rows of the disturbance matrices U and V are i · i · Normal with zero means and covariance matrices denoted by, respectively, Σ and Ω. When (1) is derived from (2) and is its reduced form, we have

$$BP + \Gamma = 0. \tag{3}$$

Also when a priori restrictions on A are available they can be used to estimate A first, and find the Derived Reduced-Form (DRF) estimators of P from (3). When such restrictions are simply exclusion restrictions we can represent them as follows:

$$s - S\alpha = \operatorname{Vec} A, \tag{4}$$

where Vec denotes stacking by rows, α is the vector of the non-zero (unrestricted) elements of A, s is a selection vector such that its ith subvector, s_i, selects the ith endogenous variable as the dependent variable of the ith equation (normalization), and S is a block-diagonal selection matrix such that its ith block, S_i, satisfies $XS_i = X_i$, the matrix of observations on the explanatory variables appearing in the ith equation.

Model (1), whether it is considered the reduced form of some other behavioral model or not, can be estimated by a variety of methods. Let $\hat{P} = (Y'Z)(Z'Z)^{-1}$ denote the Unrestricted Least Squares (ULS) and $P^+ = -B^{+-1}\Gamma^+$ any DRF estimate based on the OLS, 2SLS, 3SLS and FIML estimators of A. These five estimators are the traditional or standard methods that we consider in our comparisons. Under the classical assumptions [e.g., see Sargan (1976a)], ULS has desirable finite and large sample properties but is asymptotically less efficient than the full information P^+ based on 3SLS or FIML. $\hat{P}$ remains unbiased and consistent so long as X is properly specified even though X_i may be incorrectly specified in the ith equation. $\hat{P}$ is thought to be robust whereas P^+'s are expected to be fragile with respect to many specification uncertainties. FIML-based P^+ (like $\hat{P}$) is the only *standard* DRF estimator known to have some finite sample moments, and thus not expected to produce frequent outliers in forecasting from (1); see Sargan (1976b). 2SLS and 3SLS-based reduced-form estimators do not possess even a finite mean or variance; see McCarthy (1972) and Sargan (1976b). One must bear in mind all these properties in studying the evidence in section 4.

3. Non-standard techniques

The specification uncertainties *and* poor small sample properties of standard SEM methods have motivated the development of several alternative approaches. Approximating (linearizing) the a priori restrictions in (3) by using (e.g.) 2SLS estimates of A, one row at a time, provides the PRRF estimator, see Kakwani and Court (1972). Treating (1) as a Bayesian forecasting model and employing the a priori restrictions to partially assess prior p.d.f.'s on the coefficient P leads to the GRF model and estimation techniques; see Maasoumi (1986). Finally, MSRF is obtained as a weighted average of $\hat{P}$ and a DRF estimator with the weights depending on the outcome of a general test for the specification of the behavioral model in (2); see Maasoumi (1978).

PRRF is defined as follows:

$$\begin{aligned} \operatorname{Vec} \tilde{P} &= \tilde{p}, \quad \text{say}, \\ &= (I \otimes \hat{Q}') S \tilde{\tilde{\alpha}}, \end{aligned} \tag{5}$$

where $\tilde{\tilde{\alpha}}$ is the 2SLS (or some other) estimator of α, $\hat{Q}' = (\hat{P}' \vdots I_m)$, and (5) is derived from a row vectorization of (3) and (4).[2] This estimator has a mixed-regression interpretation and possesses finite moments; see Byron (1976) and Knight (1977), respectively.

The MSRF estimator is defined as follows:

$$p^* = \lambda p^+ + (1 - \lambda) \hat{p}, \tag{6}$$

$$\begin{aligned} \lambda &= 1 && \text{if} \quad \phi^+ \leq C_p, \\ &= (\phi_2/\phi^+)^{1/2} && \text{if} \quad \phi^+ > C_p, \quad \text{with any} \quad \phi_2 \leq C_p, \end{aligned}$$

where

$$\phi^+ = \operatorname{tr}\left[\hat{\Omega}^{-1}(\hat{P} - P^+)(Z'Z)(\hat{P} - P^+)'\right] \underset{\text{a}}{\sim} \chi^2_q. \tag{7}$$

$\hat{\Omega}$ is a consistent estimate of Ω, and C_p is the $(1 - p)$ 100 critical value of a χ^2 distribution with degrees of freedom q equal to the total degrees of over-identification in (2).

When $P^+ \equiv$ 3SLS, Maasoumi (1978) gave sufficient conditions for the MSRF to possess finite moments of order $T - n - m$ or less, and showed that it was approximately asymptotically equivalent to 3SLS. Clearly MSRF will be identical with its DRF component (P^+) if ϕ^+ does not reject the a priori restrictions. With relatively small samples, however, ϕ^+ is known to have a tendency to over-reject even correctly specified models.

The GRF model assumes that the data generating process is (1) together with:

$$\operatorname{Vec} P \sim \mathrm{N}\left(\operatorname{Vec} \bar{P}, \Omega_p\right) \quad \text{independent of } V, \tag{8}$$

$$B\bar{P} + \Gamma = 0, \tag{9}$$

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[2] Lower case letter p and its estimates denote Vec P and its estimates.

and also with the a priori restrictions in (4) which are now imposed on $\bar{P}$, the partially assessed prior mean of P. For a full discussion of GRF and the assessment of Ω_p and other issues see Maasoumi (1986). Assuming that B^{-1} exists, it may be verified that the GRF model implies and is implied by the following relations:

$$BP + \Gamma = D, \tag{10}$$

where

$$\operatorname{Vec} D \sim \mathrm{N}(0,\Omega_d) \quad \text{and} \quad \Omega_d = (B \otimes I)\Omega_p(B' \otimes I).$$

D is a random representation of several different types of structural misspecifications which violate the usual $BP + \Gamma = 0$ relations separately or jointly. As examples of such misspecifications we cite (i) violations of (4), (ii) incomplete models which, in reality, are part of a larger model, and (iii) incorrect linear specifications of non-linear relations. Maasoumi (1983, sec. 2) contains a detailed discussion of these examples.

The classical SEM approach assumes $\Omega_d = 0$. The GRF model provides a formalism for investigating generally unknown departures from this extreme assumption.

The GRF estimators, (α^*, P^*), are given as follows:

$$\alpha^* = \alpha^+ + F^+ S'(I \otimes Q^+)\Omega_d^{-1}(B^+ \otimes I)\operatorname{Vec}(P^* - P^+), \tag{11}$$

$$\operatorname{Vec} P^* = W \operatorname{Vec} \hat{P} + (I - W)\operatorname{Vec} P^+, \tag{12}$$

where

$$F^+ = \left[S'(I \otimes Q^+)\Omega_d^{-1}(I \otimes Q^{+\prime})S\right], \qquad Q^{+\prime} = \left[P^{+\prime}\ I_m\right].$$

B^+ and α^+ are the corresponding traditional SEM estimates of B and α, and W is a weight matrix given in Maasoumi (1986, eq. (20)).

In finite samples, P^* has finite rth-order moments if the equations of the maintained *behavioral* model (2) are over-identified by $2r$ or more degrees. The asymptotic properties of (α^*, P^*) depend on whether the assumed GRF model is the true model or (2) and its reduced form are the true model. In the former case [Case 1 of Maasoumi (1986)] $\Omega_d = \mathrm{O}(T^{-1})$, or smaller, is required as a characteristic of the underlying misspecifications [compare with Fisher (1961)]. The attraction of GRF is that it pulls toward its DRF component with larger sample sizes and/or greater certainty in the maintained behavioral model, and pulls toward the ULS estimator otherwise. We will witness these properties in the next section which details our numerical experience with these estimators.

4. The available numerical evidence

We have made a beginning in the comparative study of the non-traditional methods mentioned above by focusing on a subset of practically interesting situations in which these estimators may be used. One such interesting situation is an actual model which has done statistically poorly and is therefore uncertain to an unknown degree. We have used Klein's Model I of the U.S. economy as well as a small monetarist model, which is an adaptation of Stein's (1982).

Another interesting set of questions can only be answered with greater control over the accuracy of specifications or lack thereof. This necessitates Monte Carlo simulation experiments. Below we report the results of several useful experiments which are informative about the estimators *and* the influential model characteristics that require further investigation.

4.1. Estimates of two macro-models

Table 1 below describes Klein's Model I and gives the estimates of its behavioral equations. Variable definitions are given in Theil (1971). As is well known, with the 21 annual observations the ϕ^+ test described above (equalled 1082) and its variants are substantially greater than any conventional critical level of χ^2_{12}. Three sets of GRF estimates are given for different levels of Ω_d representing increasing degrees of uncertainty. A starting value for Ω_d was described in Maasoumi (1986, sec. 5) as follows:

$$\Omega_d = C^*\left(\tilde{\Sigma} \otimes (Z'Z)^{-1}\right),$$

where $\tilde{\Sigma}$ is the 2SLS residuals' estimate of Σ and C^* is an arbitrary scalar. If $\lim(Z'Z/T) = M$ is a finite matrix, then Ω_d above is $O(T^{-1})$. It can be made to have smaller orders of magnitude in T by suitable choices of C^*.

Considering the strong rejection of the model it is not surprising to find noticeable variation amongst the traditional structural estimates. FIML estimates are generally very different from the other estimates reflecting their extreme sensitivity to misspecification. The larger standard errors for FIML reflect the non-existence of its moments; see Sargan (1970). The GRFs with smaller $C^*(\Omega_d)$ are closer to 2SLS and 3SLS estimates which do have finite moments, but have smaller standard errors than the latter. The GRF estimates are only slightly closer to FIML in the case of extreme uncertainty ($C^* = 50.0$).

The reduced-form coefficient estimates are given in table 1 of Maasoumi (1986). Table 2 below reports some well-known statistics bearing on the in-sample predictive performance of all the estimators. The entries on the GRF are different here from those reported in tables 1–3 of Maasoumi (1986)

Table 1

Structural coefficient estimates under specification uncertainty.[a]

Klein's Model I of the U.S. economy (1921–1941)

(i)	Consumption	$C = a_0 + a_1 P + a_2 P_{-1} + a_3 W + e_1$	(iv)	Product	$Y = C + I + G - T$
(ii)	Investment	$I = b_0 + b_1 P + b_2 P_{-1} + b_3 K_{-1} + e_2$	(v)	Profits	$P = Y - W^* - W^{**}$
(iii)	Private wages	$W^* = c_0 + c_1 E + c_2 E_{-1} + c_3 t + e_3$	(vi)	Capital stock	$K = K_{-1} + I$
			(vii)	Private product	$E = Y + T - W^{**}$

			Consumption equation				Investment equation				Private wages equation			
	$\mathrm{tr}\,\Omega_d$	$\mathrm{tr}\,\Omega_p$	1	P	P_{-1}	W	1	P	P_{-1}	K_{-1}	1	E	E_{-1}	t
2SLS	0.0	0.0	−16.55 (1.32)	0.017 (0.118)	0.216 (0.107)	0.810 (0.040)	20.28 (7.54)	0.150 (0.173)	0.616 (0.163)	−0.158 (0.036)	1.500 (1.15)	0.439 (0.035)	0.147 (0.039)	0.130 (0.029)
3SLS	0.0	0.0	16.44 (1.30)	0.125 (0.108)	0.163 (0.100)	0.790 (0.038)	28.18 (6.79)	−0.013 (0.162)	0.756 (0.152)	−0.195 (0.033)	1.797 (1.11)	0.400 (0.032)	0.181 (0.034)	0.150 (0.028)
FIML	0.0	0.0	18.34 (2.84)	−0.232 (0.036)	0.386 (0.217)	0.802 (0.036)	27.26 (7.89)	−0.801 (0.491)	1.052 (0.352)	−0.148 (0.029)	5.793 (1.80)	0.234 (1.049)	0.285 (0.45)	0.234 (0.34)
GRF														
$C^* = 0.05$	0.236	2.43	16.31 (25.9)	0.167 (0.010)	0.110 (0.010)	0.797 (0.004)	25.89 (0.165)	0.147 (0.014)	0.650 (0.014)	−0.188 (0.003)	1.273 (1.52)	0.425 (0.005)	0.165 (0.005)	0.133 (0.004)
$C^* = 1.0$	0.387	3.04	16.20 (25.9)	0.237 (0.045)	0.089 (0.046)	0.779 (0.178)	29.29 (0.300)	0.043 (0.062)	0.716 (0.061)	−0.202 (0.015)	1.866 (0.161)	0.392 (0.022)	0.188 (0.025)	0.158 (0.019)
$C^* = 50.0$	1.81	6.45	16.31 (25.8)	0.189 (0.300)	0.110 (0.120)	0.788 (0.119)	30.08 (1.43)	0.003 (0.468)	0.749 (0.460)	−0.205 (0.107)	1.68 (11.2)	0.410 (0.096)	0.272 (0.108)	0.146 (0.087)

[a] Standard errors of the estimates in parentheses.

Table 2

Actual and fitted value comparison of reduced-form models.[a]

Klein's Model I

(1) Consumption equation

		DRF						GRF				
	ULS	OLS	2SLS	3SLS	FIML	PRRF	MSRF	$C^* = 0.05$	$C^* = 0.5$	$C^* = 1.0$	$C^* = 5.0$	$C^* = 50.0$
RMSE	1.663	2.803	1.981	1.963	2.282	1.790	1.667	1.886	1.768	1.744	1.694	1.664
	(1)	(12)	(10)	(9)	(11)	(7)	(3)	(8)	(6)	(5)	(4)	(2)
MAE	1.360	2.048	1.619	1.655	1.858	1.505	1.360	1.483	1.396	1.389	1.338	1.356
	(3)	(12)	(9)	(10)	(11)	(8)	(3)	(7)	(6)	(5)	(1)	(2)
Slope coefficient	1.000	0.850	0.983	1.004	1.079	1.000	1.003	1.037	1.018	1.016	0.999	1.002
Theil's U-statistic												
Total	0.015	0.026	0.018	0.018	0.021	0.016	0.015	0.017	0.016	0.016	0.016	0.015
	(1)	(6)	(4)	(4)	(5)	(2)	(1)	(3)	(2)	(2)	(2)	(1)
(a) bias	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
(b) variation	0.016	0.041	0.009	0.027	0.137	0.019	0.019	0.070	0.039	0.037	0.016	0.018
(c) covariation	0.984	0.959	0.991	0.973	0.863	0.981	0.981	0.930	0.961	0.963	0.984	0.982

[a] The numbers in brackets are the ranking of the corresponding estimator.

and are thus additional evidence on the robust performance of GRF.[3] Not surprisingly, the Unrestricted Least Squares (ULS) is generally the 'best' estimator, specially according to Root Mean Squared Error (RMSE) and Theil's U-statistic. The Mean Absolute Error (MAE) and the slope of the regression of the predicted values and the actual observations are also reported. The GRF estimator performs extremely well specially with more appropriate levels of uncertainty (larger C^*). The GRF estimates here are generally closer to the ULS than the other estimators, and closer than the GRF with smaller C^* used in tables 1–3 of Maasoumi (1986). As expected, specification of smaller values for C^* moves GRF closer to its DRF component. Thus while all the non-traditional estimators dominate all the traditional methods, the PRRF and the MSRF are generally third or second only to the GRF based on a Ω_d which properly reflects the inadequacy of the model. Furthermore, MSRF and PRRF are also very close to ULS in these circumstances [see (7)] and perform specially well according to the MAE criterion (except for PRRF applied to the consumption equation). We conclude that in this example there is no loss in applying the non-traditional methods, and there are some noticeable gains in using the GRF since it also provides estimates for the behavioral equations.

Jeong (1985, ch. VI) contains the full details of a three-equation, dynamic, quarterly model of unemployment and inflation. This model was developed on the basis of extensive data analysis, including statistical searches and tests of 'causality', and is statistically a more appealing variant of the monetarist model described in Stein (1982). Utilizing a larger sample (U.S. data for 1961.I–1983.IV), the model is more reliably tested by asymptotically valid procedures than Klein's Model I with its small sample of 21 annual observations.

Jeong (1985) found general confirmation for the observations made above. One major difference here is that medium values of $C^*(\Omega_d)$ produce predictably better GRF estimates than the extreme values which performed best for Klein's model. Table 3 presents the behavioral model and its coefficients. M denotes the rate of money growth and its lagged values. Table 4 has the same design as table 2. Similar statistics for the inflation rate and MFP reduced-form equations are available from the authors. ULS and GRF are the best performers even though the traditional *full information* estimators do better than in the previous example.

We also compared the performance of the 3SLS-based with FIML-based GRF in both of the above models. In the statistically less satisfactory model of

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[3] In Maasoumi (1986) all the values for C should be divided by $\sqrt{21} \doteq 4.6$. Consequently, tables 1–3 of that paper report on GRF with $C^* \doteq (0.01, 0.1, 0.2 \text{ and } 10)$. In the current report $C^* = (0.05, 0.5, 1.0, 5.0 \text{ and } 50)$ are the values which underly our conclusions. Tables similar to table 2 summarize fully conformable evidence for the investment and private wages equations. These were removed at the editor's suggestion but are available upon request.

Table 3

Structural coefficient estimation; empirical monetarist model of the U.S. economy; 1961.I–1983.IV.[a]

(1) Unemployment rate U: $U + a_0 + a_1 MFP + a_2 U_{-1} + a_3 U_{-2} + a_4 MFP_{-1} = e_1$

(2) Inflation rate P: $P + b_0 + b_1 MFP + b_2 P_{-1} + b_3 P_{-3} + b_4 MFP_{-1} = e_2$

(3) Expected real money growth rate MFP: $MFP + c_0 + c_1 P + c_2 P_{-4} + c_3 MFP_{-1} + c_4 M_{-1} = e_3$

	Unemployment					Inflation					Expected real money growth				
	1	MFP	U_{-1}	U_{-2}	MFP_{-1}	1	MFP	P_{-1}	P_{-3}	MFP_{-1}	1	P	P_{-4}	MFP_{-1}	M_{-1}
	−0.222	0.032	−1.59	0.619	0.018	−0.732	0.109	−0.601	−0.268	−0.087	−2.76	1.673	−0.341	0.567	−0.898
2SLS	(2.26)	(2.63)	(21.3)	(8.11)	(2.53)	(1.57)	(1.25)	(6.73)	(3.12)	(2.05)	(3.02)	(5.96)	(1.79)	(3.00)	(3.80)
	–	[1.79]	[1.96]	[1.96]	[1.95]	–	[1.65]	[1.93]	[1.89]	[1.94]	–	[1.89]	[1.94]	[1.94]	[1.93]
	−0.230	0.031	−1.57	0.603	0.018	−0.820	0.124	−0.571	−0.283	−0.081	−3.18	1.708	−0.446	0.466	−0.752
3SLS	(2.34)	(2.62)	(21.2)	(7.95)	(2.60)	(1.78)	(1.46)	(7.09)	(3.71)	(1.95)	(3.51)	(6.19)	(2.60)	(2.61)	(3.38)
	−0.238	0.032	−1.57	0.601	0.018	−0.345	0.006	−0.609	−0.323	−0.086	−2.90	1.759	−0.437	0.537	−0.862
FIML	(2.44)	(2.51)	(20.9)	(7.79)	(2.63)	(0.64)	(0.05)	(6.57)	(3.71)	(1.92)	(3.12)	(5.52)	(2.10)	(2.95)	(3.76)
GRF	−0.220	0.032	−1.57	0.603	0.018	−1.197	0.211	−0.550	−0.242	−0.078	−3.38	1.642	−0.354	0.473	−0.741
$C^* = 0.5$	(0.03)	(22.5)	(173.0)	(64.3)	(21.7)	(0.38)	(20.2)	(54.3)	(27.0)	(15.3)	(2.03)	(49.5)	(15.9)	(23.4)	(30.0)
	−0.222	0.032	−1.57	0.601	0.019	−1.20	0.211	−0.545	−0.247	−0.077	−3.39	1.647	−0.371	0.461	−0.726
$C^* = 1.0$	(0.03)	(5.14)	(39.5)	(14.6)	(4.98)	(0.38)	(4.61)	(12.3)	(6.27)	(3.46)	(2.04)	(11.3)	(3.81)	(5.22)	(6.70)
	−0.226	0.032	−1.57	0.597	0.019	−1.20	0.029	−0.535	−0.257	−0.075	−3.41	1.659	−0.407	0.440	−0.699
$C^* = 5.0$	(0.03)	(1.66)	(12.7)	(4.70)	(1.61)	(0.38)	(1.47)	(3.90)	(2.11)	(1.09)	(2.05)	(3.69)	(1.35)	(1.60)	(2.08)

[a] Asymptotic t-values in parentheses and Nagar type approximation of 5% significance level t-values in square brackets.

Table 4

Actual and fitted value comparisons; empirical monetarist model; 1961.I–1983.IV.

(1) Unemployment equation

		DRF						GRF				
	ULS	OLS	2SLS	3SLS	FIML	PRRF	MSRF	$C^* = 0.05$	$C^* = 0.5$	$C^* = 1.0$	$C^* = 5.0$	$C^* = 50.0$
RMSE	0.259 (1)	0.266 (6)	0.266 (6)	0.265 (5)	0.265 (5)	0.264 (4)	0.265 (5)	0.265 (5)	0.265 (5)	0.263 (3)	0.260 (2)	0.259 (1)
MAE	0.193 (1)	0.204 (7)	0.204 (7)	0.204 (7)	0.202 (5)	0.203 (6)	0.204 (7)	0.204 (7)	0.204 (7)	0.201 (4)	0.195 (3)	0.194 (2)
Slope coefficient	1.000	0.995	0.994	0.997	0.996	1.000	0.997	0.998	0.998	0.999	1.000	0.999
Theil's *U*-statistic												
Total	0.021	0.021	0.021	0.021	0.021	0.021	0.021	0.021	0.021	0.021	0.021	0.021
(a) bias	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
(b) variation	0.006	0.002	0.002	0.003	0.003	0.006	0.003	0.004	0.004	0.005	0.006	0.005
(c) covariation	0.994	0.998	0.998	0.997	0.997	0.994	0.997	0.996	0.996	0.995	0.994	0.995

Klein's, 3SLS-based GRF out-performs the more sensitive FIML-based estimator. This limited evidence is in conformity with the known theoretical properties of the 3SLS-based GRF [see Maasoumi (1986, secs. 3–4)].

4.2. The Monte Carlo simulation results

Four experimental designs are considered below on the basis of two simultaneous equations models, each having two equations and four exogenous variables. These designs are used to study (i) the effect of changes in such key design parameters as the sample size, concentration matrices and degrees of over-identification and (ii) controlled misspecifications. We also use these experiments to analyze out-of-sample forecasting performance of the competing techniques.[4]

Table 5 describes models A and B and the changes across the four experiments in each case. One equation in model B is over-identified by two degrees, the other is exactly identified. Equations of model A are over-identified by one degree each.

250 replications were based on *identical* normally distributed random disturbances for all experiments. The disturbances were generated to have zero means and the covariance matrix (Ω) given in table 5.

Experiments I and II consider only the effect of a change in the sample size. Experiments I and III should be compared to evaluate the effect of changes in the centrality (concentration) matrices. Finally, experiments I and IV should be compared to trace the effects of *erroneous* omission and inclusion of exogenous variables. When data generated by model A are used to estimate model B, the first equation of A suffers from erroneously included variables, and its second equation has an omitted variable misspecification. The situation is reversed for the equations of model B.

Structural estimates – Tables B1–B2 of Maasoumi and Jeong (1986, app. B) report the traditional as well as two variants of the GRF estimates of the structural coefficients of models A and B. In the interest of brevity we only summarize the most striking features of the results:

(a) The known inconsistency of OLS is quite evident. It is very tightly distributed around a very poor estimate suggesting that variance alone will give a misleading ranking for it.

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[4] Given the incremental nature of learning we were forced to choose between a more extensive study of a limited number of designs and a limited analysis of an extensive set of designs. We chose the former for now. This first step makes sense in that we have also learned what other designs deserve to be investigated in the future.

(b) Larger sample size improves all the structural estimates and makes them much more concentrated around the true values.

(c) Model A is somewhat better estimated than model B. A just identified equation in the latter suggests the non-existence of finite moments for all but the OLS estimator.

(d) Reduction in the centrality coefficients leads to a marked deterioration in estimator performance, especially for the estimated coefficient of the exogenous variable.

(e) In almost all cases in experiments I–III the GRF structural estimator does at least as well as the traditional limited and full information estimators. This may be surprising since in these experiments the models are correctly specified. While FIML structural estimates perform relatively badly because they do not possess finite moments [see Sargan (1970)], 2SLS and 3SLS may do better in more highly identified models [see Sargan (1978)].

(f) Misspecifications of experiment IV more adversely affect model B estimates. In particular, FIML is the most dispersed estimator with this type of *omitted variable* misspecification. The erroneously included variable misspecification of model A does cause a little more bias, the most for FIML and the least for 2SLS, reflecting the relative sensitivities of these estimators to misspecification in SEMs. However, increased dispersion is the most dramatic effect of this kind of misspecification.

Reduced-form estimates – Table 6 reports the estimated reduced-form coefficient of Z_3 in the equation for Y_1. Z_3 is included in model B, but excluded from the corresponding equation of model A. Z_3 is also the instrument of misspecifications in experiment IV. Maasoumi and Jeong (1986) have similar tables for other coefficients. The salient features of the results may be summarized as follows:

(a) In experiments I and II, the two GRFs and the MSRF estimators generally perform at least as well as the best of the traditional estimators. There is quite clearly no loss in applying these estimators in situations that are most favorable to traditional SEM estimators (with small or larger samples). The latter perform extremely well in experiments I and II.

(b) ULS and PRRF are ranked last in experiments I and II on the basis of bias and dispersion. They are often identical because of the exact identification of one equation in model B and the near identity of the systematic parts of Y_1 in models A and B. While ULS does not perform badly in absolute terms, its relative inefficiency is demonstrated.

(c) Experiment III has a smaller concentration matrix than experiment I. More importantly $Z'Z$ is no longer $O(T)$ in this experiment which could have a serious effect on the properties of estimators, particularly their asymptotic properties. Consequently, there is a clear deterioration in the

Table 5
Two experimental models.

	Model A	Model B
(a) $A = [B : \Gamma] =$	$\begin{bmatrix} 1 & -0.5 : 0.5 & -0.75 & 0 & 0 \\ -4 & 1 : 0 & 0 & 4.0 & -1.6 \end{bmatrix}$	$\begin{bmatrix} 1 & -0.5 : 0.5 & -0.75 & 4.0 & 0 \\ -4 & 1 : 0 & 0 & 0 & -1.6 \end{bmatrix}$
Σ and $\Omega =$	$\begin{bmatrix} 1.0 & 0.5 \\ 0.5 & 1.0 \end{bmatrix}$ $\begin{bmatrix} 1.75 & 6.0 \\ 6.0 & 21.0 \end{bmatrix}$	Same as model A
(b) $P =$	$\begin{bmatrix} 0.5 & -0.75 & 2.0 & -0.8 \\ 2.0 & -3.0 & 4.0 & -1.6 \end{bmatrix}$	$\begin{bmatrix} 0.5 & -0.75 & 4.0 & -0.8 \\ 2.0 & -3.0 & 16.0 & -1.6 \end{bmatrix}$
Experiment I ($T = 24$, *higher centrality, correct specification*)		
(a) $\|Z'Z\|$	$\|Z'Z\| = \|24 I_4\| = 24^4$	Same as model A
(b) Degree of overidentification $\upsilon = \upsilon_1 + \upsilon_2$	$2 = 1 + 1$	$2 = 0 + 2$
(c) Non-centrality parameter $P(Z'Z/T)P'$	$\begin{bmatrix} 5.45 & 12.53 \\ 12.53 & 31.56 \end{bmatrix}$	$\begin{bmatrix} 17.45 & 68.54 \\ 68.54 & 271.59 \end{bmatrix}$
(d) $\mathrm{tr}(\Omega^{-1}(PZ'ZP')/T$	25.83	25.83
Experiment II ($T = 48$, *higher centrality, correct specification*)		
(a) $\|Z'Z\|$	$\|Z'Z\| = \|48 I_4\| = 48^4$	Same as model A

	Experiment III ($T = 24$, *lower centrality*, *correct specification*)	
(a) $\lvert Z'Z \rvert$	$\lvert Z'Z \rvert = \lvert I_4 \rvert = 1.0$	Same as model A
(c) $PZ'ZP'/T$	$\begin{bmatrix} 0.22 & 0.52 \\ 0.52 & 1.32 \end{bmatrix}$	$\begin{bmatrix} 0.73 & 2.86 \\ 2.86 & 11.30 \end{bmatrix}$
(d) $\operatorname{tr}(\Omega^{-1}(PZ'ZP')/T$	1.076	1.076
	Experiment IV ($T = 24$, *higher centrality*, *misspecification*)	
Perceived structural model	Model B	Model A
Exogenous variables based on		
$Z' \quad = l_i' \otimes \begin{bmatrix} 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ \sqrt{2} & 0 & -\sqrt{2} & 0 & \sqrt{2} & 0 & -\sqrt{2} & 0 \\ 0 & \sqrt{2} & 0 & -\sqrt{2} & 0 & \sqrt{2} & 0 & -\sqrt{2} \end{bmatrix}$		$= l_i' \otimes Z^{*\prime}$
l_ι = i-element vector of ones	$\bar{Z}_j^* = 0,\ j = 1,\dots,4$	$Z^{*\prime}Z^* = TI_4$

Table 6

Distribution of the reduced-form coefficient of Z_3; true value for model A = 2.00 and true value for model B = 4.00.

	Mean	Std. dev.	Skew-ness	Kur-tosis	Median	Length of interval		
						50%	80%	100%
Experiment I, model A, correct specification, sample size = 24								
ULS	1.965	0.257	0.29	0.83	1.97	0.32	0.64	1.70
DRF-2SLS	2.015	0.241	0.10	0.92	2.01	0.32	0.56	1.69
DRF-3SLS	2.026	0.243	0.04	0.78	2.03	0.32	0.58	1.69
DRF-FIML	1.976	0.240	0.16	0.94	1.97	0.30	0.59	1.69
PRRF	1.918	0.308	−0.10	0.76	1.94	0.34	0.75	2.03
MSRF	2.005	0.244	0.17	0.85	2.00	0.31	0.57	1.69
GRF (C^* = 1.0)	1.974	0.241	0.19	0.98	1.97	0.30	0.59	1.69
GRF (C^* = 0.05)	1.976	0.240	0.16	0.94	1.97	0.30	0.59	1.69
Experiment I, model B, correct specification, sample size = 24								
ULS	3.965	0.257	0.29	0.83	3.97	0.32	0.64	1.70
DRF-2SLS	3.976	0.254	0.27	0.74	3.97	0.31	0.64	1.65
DRF-3SLS	3.987	0.253	0.26	0.64	3.98	0.32	0.64	1.61
DRF-FIML	3.969	0.253	0.27	0.63	3.97	0.32	0.64	1.59
PRRF	3.965	0.257	0.29	0.83	3.97	0.32	0.64	1.70
MSRF	3.987	0.253	0.26	0.64	3.98	0.32	0.64	1.61
GRF (C^* = 1.0)	3.970	0.253	0.27	0.67	3.96	0.31	0.63	1.61
GRF (C^* = 0.05)	3.971	0.253	0.27	0.63	3.97	0.32	0.65	1.59
Experiment II, model A, correct specification, sample size = 48								
ULS	1.966	0.173	−0.01	0.32	1.97	0.21	0.44	1.02
DRF-2SLS	1.991	0.165	−0.08	0.34	1.99	0.20	0.41	0.95
DRF-3SLS	1.997	0.167	−0.09	0.25	2.00	0.21	0.43	0.94
DRF-FIML	1.969	0.165	−0.01	0.34	1.97	0.21	0.41	0.96
PRRF	1.942	0.202	−0.26	0.45	1.95	0.23	0.49	1.19
MSRF	1.988	0.165	−0.04	0.34	1.99	0.20	0.42	0.94
GRF (C^* = 1.0)	1.969	0.164	−0.01	0.37	1.98	0.20	0.41	0.96
GRF (C^* = 0.05)	1.969	0.164	−0.01	0.34	1.98	0.21	0.41	0.96
Experiment II, model B, correct specification, sample size = 48								
ULS	3.992	0.185	−0.01	−0.22	3.98	0.27	0.46	0.99
DRF-2SLS	3.998	0.182	−0.00	−0.12	3.99	0.26	0.46	0.97
DRF-3SLS	4.003	0.180	−0.00	−0.01	3.99	0.26	0.46	1.00
DRF-FIML	3.994	0.181	0.02	−0.06	3.98	0.27	0.45	1.01
PRRF	3.992	0.185	−0.01	−0.22	3.98	0.27	0.46	0.99
MSRF	4.003	0.180	−0.00	−0.06	3.99	0.26	0.46	1.00
GRF (C^* = 1.0)	3.994	0.181	0.02	−0.08	3.98	0.27	0.45	1.00
GRF (C^* = 0.05)	3.994	0.181	0.02	−0.06	3.98	0.27	0.45	1.01
Experiment III, model A, correct specification, lower centrality, sample size = 24								
ULS	1.828	1.258	0.29	0.83	1.85	1.58	3.12	8.31
DRF-2SLS	14.408	147.708	14.69	223.56	2.68	2.48	7.48	2320.95
DRF-3SLS	8.138	83.762	15.60	245.41	2.59	2.06	5.65	1342.27
DRF-FIML	2.559	11.103	15.51	243.73	1.87	1.44	3.07	179.99
PRRF	1.460	1.442	0.36	0.15	1.57	1.97	3.66	8.45
MSRF	7.445	83.606	15.71	247.80	2.19	1.72	4.71	1342.27
GRF (C^* = 1.0)	1.858	1.255	0.04	0.92	1.90	1.53	3.03	9.08
GRF (C^* = 0.05)	1.863	1.257	0.13	0.64	1.87	1.61	2.98	8.50

Table 6 (continued)

						Length of interval		
	Mean	Std. dev.	Skewness	Kurtosis	Median	50%	80%	100%
Experiment III, model B, correct specification, lower centrality, sample size = 24								
ULS	3.828	1.258	0.29	0.83	3.85	1.58	3.12	8.31
DRF-2SLS	3.758	4.716	−11.70	163.64	4.04	1.66	3.22	78.28
DRF-3SLS	4.132	1.609	−3.96	35.84	4.23	1.52	2.97	20.08
DRF-FIML	3.814	1.255	0.19	0.89	3.80	1.58	3.19	8.97
PRRF	3.828	1.258	0.29	0.83	3.85	1.58	3.12	8.31
MSRF	4.198	1.172	0.26	1.10	4.19	1.48	2.94	8.88
GRF ($C^* = 1.0$)	3.887	1.202	0.35	0.89	3.86	1.54	2.95	8.24
GRF ($C^* = 0.05$)	3.892	1.205	0.32	0.89	3.89	1.57	2.87	8.33
Experiment IV, model A, misspecified as model B, sample size = 24								
ULS	1.965	0.257	0.29	0.83	1.97	0.32	0.64	1.70
DRF-2SLS	0.259	26.439	0.59	69.57	0.85	2.31	12.36	498.63
DRF-3SLS	1.235	15.210	1.30	73.69	1.54	1.54	6.27	295.35
PRF-FIML	1.722	1.003	0.24	5.34	1.91	1.04	2.77	8.57
PRRF	1.965	0.257	0.29	0.83	1.97	0.32	0.64	1.70
MSRF	1.251	15.210	1.29	73.67	1.57	1.50	6.27	295.35
GRF ($C^* = 1.0$)	2.264	0.409	−0.03	−0.38	2.30	0.56	1.04	1.97
GRF ($C^* = 0.05$)	2.254	0.423	0.35	−0.25	2.21	0.63	1.09	2.21
Experiment IV, model B, misspecified as model A, sample size = 24								
ULS	3.965	0.257	0.29	0.83	3.97	0.32	0.64	1.70
DRF-2SLS	9.925	71.76	15.12	234.89	4.31	2.38	6.53	1182.93
DRF-3SLS	9.077	62.217	15.12	237.02	4.26	1.89	5.12	1023.27
PRF-FIML	2.642	4.306	0.40	57.71	3.01	2.50	3.88	76.45
PRRF	4.009	0.256	0.26	0.88	4.02	0.32	0.64	1.73
MSRF	3.969	0.256	0.27	0.78	3.97	0.32	0.63	1.68
GRF ($C^* = 1.0$)	3.968	0.439	0.35	−0.27	3.93	0.69	1.08	2.36
GRF ($C^* = 0.05$)	4.007	0.385	0.58	1.26	3.98	0.47	0.87	2.55

performance of all estimators in experiment III. 2SLS is the most dramatically affected and is now the worst performer by every measure reported in table 6 *except for the median*. The MSRF estimator is close to the best estimator (GRF), particularly in model B. The asymptotic χ^2 test generally fails to reject model A or B. Consequently, MSRF will be almost identical with the 3SLS reduced-form estimator. When the latter performs badly, as in the case of Z_3 in model A, it adversely affects the MSRF which in view of lack of precision in the data (small $Z'Z$) is inconsistent and over-dispersed. MSRF never performs worse than the 3SLS estimator, however!

The ULS and PRRF perform reasonably well but generally not as well as the two GRF estimators.

(d) In experiment IV the DRF estimators (2SLS, 3SLS and to a lesser extent FIML) perform very poorly and should not be used. In the case of model

A, misspecification reduces the equation to a just identified relation at which point the χ_q^2 specification test generally fails to reject the model. This brings the MSRF estimator very close to the 3SLS estimator which does not perform well. The reverse happens in model B where exclusion of Z_3 increases the degree of identification of the equation. The χ^2 test decisively rejects the model and, as expected, pulls the MSRF close to the ULS estimator.

It is reassuring to observe the strong performance of the ULS which ignores the misspecified structural information. PRRF does correspondingly well, but we think this is partly due to the design of the models here. Both of the GRF estimators perform extremely well, especially in model B and for the larger value of C^* that more properly reflects the inadequacy of specification in this experiment.

Forecasting – Table 7 gives the ranking of the eight estimators in four experimental cases for models A and B. The corresponding cumulative rankings are quite revealing. Further details on these *forecast error distributions* are given in tables B3–B6 of Maasoumi and Jeong (1986). Our general reading, particularly from the cumulative rankings, is as follows:

(a) In reasonably well specified models we would not use any of the traditional SEM methods. Even though FIML occasionally performs well for correctly specified models, it is almost always out-performed by a GRF estimator. These results are consistent with the known properties of SEM reduced-form estimators, see Sargan (1976b).

(b) Among the non-traditional methods PRRF performs the worst for reasonably specified models in spite of having finite moments. In fact both 2SLS and 3SLS predictors out-perform it in correctly specified models. Only ULS does slightly worse than PRRF in similar circumstances.

(c) GRF estimates are best under RMSE and generally so under the probability range criterion. For most correctly specified models the margin of advantage is undeniable. ULS (PRRF) is, however, the best performer under misspecification (according to RMSE or the distribution range). Under misspecification (case IV) all the new methods out-perform all of the traditional SEM methods. Because of uncertainties regarding the existence and the degrees of misspecifications in practical situations we recommend the GRF or the MSRF over the ULS forecasts.

5. Conclusion

While we recognize the specificity of our findings and, in particular, the dependence of forecast errors on other factors than just the estimation errors, we find the current evidence a further confirmation of the known theoretical properties of the GRF and the MSRF estimators.

Table 7

Ranking of forecasts.[a]

	Case I			Case II			Case III			Case IV			
nc	Model A $Y_1 Y_2$	Model B $Y_1 Y_2$	Total	Model A $Y_1 Y_2$	Model B $Y_1 Y_2$	Total	Model A $Y_1 Y_2$	Model B $Y_1 Y_2$	Total	Model A $Y_1 Y_2$	Model B $Y_1 Y_2$	Total	Grand total
					RMSE (std. dev. of samples) rankings								
ULS	7 7	7 7	28	7 7	7 7	28	3 3	5 5	16	1 2	2 2	7	79
DRF-2SLS	6 6	6 6	24	6 6	6 6	24	8 8	8 8	32	8 8	8 8	32	112
DRF-3SLS	5 3	2 2	12	4 4	2 2	12	7 7	7 7	28	6 7	7 7	27	79
DRF-FIML	3 3	4 5	15	2 2	1 2	7	5 5	1 1	12	5 5	6 6	22	56
PRRF	8 8	7 8	31	8 8	7 8	31	4 4	5 6	19	1 1	2 3	7	72
MSRF	3 5	2 2	12	5 5	2 2	14	6 6	4 4	20	6 6	1 1	14	60
GRF ($C^* = 1.0$)	2 2	1 1	6	3 3	2 1	9	1 1	2 2	6	3 3	5 5	16	37
GRF ($C^* = 0.05$)	1 1	4 4	10	1 1	2 5	9	2 2	3 3	10	4 4	4 4	16	45
					Quintile (80% range) length rankings								
ULS	7 7	7 7	28	7 7	8 7	29	3 3	6 6	18	1 2	1 1	5	82
DRF-2SLS	6 6	6 6	24	5 4	6 4	19	8 8	8 8	32	8 8	8 8	32	107
DRF-3SLS	3 5	1 3	12	4 5	2 5	16	7 7	4 5	23	6 6	7 7	26	78
DRF-FIML	3 2	5 1	11	2 1	2 2	7	1 1	1 1	4	5 5	6 6	22	46
PRRF	8 8	7 8	31	8 8	1 8	25	5 5	6 7	23	1 1	2 3	7	86
MSRF	5 4	1 3	13	6 6	6 5	23	6 6	4 4	20	6 6	3 2	17	70
GRF ($C^* = 1.0$)	1 2	3 5	11	2 3	2 2	9	2 2	1 2	7	3 3	5 5	16	43
GRF ($C^* = 0.05$)	2 1	4 2	9	1 2	2 1	6	4 4	3 3	14	4 4	4 4	16	45

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[a] The forecast period values of exogenous variables were $(1\ 1\ \sqrt{2}\ 0)$.

Several important questions deserve further study. Among these, simulation experimentation with richer models than ours and practical forecasting with medium-size models seem the most important.[5]

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[5] Two CDC software packages, one by each of the authors, are available at Indiana University which allow estimation and simulation experimentations of the type reported here.