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MULTIDIMENSIONAL ANALYSIS OF INEQUALITY IN IRAN

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ABSTRACT

We demonstrste a multidimensional approach for welfare and inequality analysis. This is the "two step" method of Maasoumi(1986) which is also a main method for the implementation of Amartya Sen's "capabilities" approach to welfare measurement. A composite measure of several welfare attributes is proposed which has also been derived in several other, more direct, axiomatic approaches. This produces a composite welfare function that combines several attributes together. Inequality in the proposed composite measures is computed using relative inequality indexes including the Generalized Entropy(GE) measures. The analysis is performed on Iran data with use of Iranian householdes income study.

Key words: Aggregate function; Composite function; Generalized entropy; Income; Number of rooms; Building area; Home appliances; Home utilities.

1. INTRODUCTION

Income inequality and income distribution across the population of an economy are unquestionably of crucial importance to a full understanding of the impact of politically and secularly induced economic change. In the traditional analyses of inequality, individuals are characterized and ordered by a specific set of welfare attributes (e.g. income) in which they share. Today, there is general agreement among economics that the traditional money-income measures are inadequate and more comprehensive measures of economic status are needed. Serious attempts have been made to analyze distributional issues in a broader contex than measured income approaches. Shorrocks (1983), Fei, Ranis, and Kuo (1978), Kakwani (1980), and Pyatt, Chen, and Fei (1980) presented factor-component analyses of income. The effect of combining welfare indicators such as wealth and net worth with income in composite measures of economic welfare was studied by Weisbrod and Hansen (1968), Murray (1964), and Projector and Weiss (1969) .In the multi-dimensional contex, Kolm (1977) analyzed the properties of the social welfare functions (SWF's) and the corresponding ranking of distributions. Atkinson and Bourguignon (1982) analyzed the conditions of first and second-order stochastic dominance of multivariate distributions.

More recent discussion of this kind of approach is found in Maasoumi (1986), Maasoumi and Nickelsburg (1988), Atkinson and Bourguignon (1982), Kakwani (1984), and Rosen (1984). Maasoumi (1986) developed an approach based on information theory. There are two steps in this approach. The first is to find an aggregate or composite measure of well-being. The second step is to apply a suitable measure of univariate inequality, say, to the aggregated measure. The choice of a measure in the second step will be guided by axiomatic approach developed by Bourguignon (1979) and Shorrocks (1980, 1984) which identify the Generalized Entropy measures (GE) as the desirable scale invariant family of inequality measures. This approach has found applications in several areas, including the Michigan Panel study of income dynamics(PSID) data on all income sources, housing equity, and education, with grouping of the sample by age and sex, see Maasoumi and Nickelsburg (1988). In this article we apply the method of Maasoumi(1986) on Iran data with use of Iranian households income study. Following the developments by

Maasoumi, we employ composite measures of well-being that combine any number of indicators in their original measured form without any conversions. In addition, our measures permit different valuations(weights) for different indicators, and admit substitution between them. Moreover, the available data may be based on indexed, categorized, or directly measured observations. Neither of these features may be generally found in such measures of well-being as those proposed by Weisbrod and Hansen(1968). The method of principal components(PC) proposed by Ram(1982) is also a special case of this approach.

The plan of this article is as follows: section 2 introduces the notation and the composite welfare measures. Section 3 presents the inequality measures and their properties. Section 4 presents our data and report of the results.

2. THE AGGREGATE OF WELFARE ATTRIBUTES

The idea that different indicators of economic welfare are distributed very differently has been supported in many studies(e.g., Lydall and Lansing 1959; Projector and Weiss 1966). Without this diversity in distributions there would be no need for a multidimensional measure. Indeed our measures of multivariate welfare also depend on this diversity. When such diversity exists, we seek a composite(scalar) measure that is a function of the welfare indicators of concern to an analyst. Maasoumi(1986) is concerned with measuring the relative inequality in the distribution of the composite measure, and thus he will propose indexes with distributions that most closely represent the distributional information in each of their constituent variables as follows:

Let X_{if} denote the amount of attribute f(f=1,2,…,M) received by individual consuming unit i(i=1,2,…,N) and $X_i = (X_{i1}, X_{i2}, ..., X_{iM})'$ be the *i*th row of the N×M matrix *X*, and $X^f = (X_{1f}, X_{2f}, ..., X_{Nf})'$. The " summary " or " aggregate " attribute function representing individual welfare is denoted by $S_i = h(X_i)$. He use generalized " information functions " which include the logarithm as a special case. This leads his to the notion of ϕ-entropy(see Burbea and Rao, 1982a, 1982b). In turn, he generalized the corresponding, pairwise, criteria of divergence in order to deal with the M$\geq$2 distributions of the X^{f}'s. The result is the following measure of divergence or expected information:

$$D_\beta(S,X;\alpha)=\sum_{f=1}^{M}\alpha_f\left\{\frac{1}{\beta(\beta+1)}\sum_{i=1}^{N}S_i\left[(\frac{S_i}{X_{if}})^\beta-1\right]\right\}, \qquad \beta\neq 0,-1\,, \qquad (1a)$$

$$=\sum_f\alpha_f\left\{\sum_i S_i\log(\frac{S_i}{X_{if}})\right\}, \qquad \beta=0\,, \qquad (1b)$$

$$=\sum_f\alpha_f\left\{\sum_i X_{if}\log(\frac{X_{if}}{S_i})\right\}, \qquad \beta=-1\,. \qquad (1c)$$

Where α_f is the analyst's valuations (weight) for the fth variable.

Note that β=-1 and 0 correspond to the(weighted sums of) directional Kullback-Leibler measures of distributional divergence. The term in {} were shown in Burbea and Rao(1982b) to be generally convex in S_i. The distribution $S=(S_1,S_2,...,S_N)$ which minimizes $D_\beta(\,)$ is as follows:

$$S_i\propto\left[\sum_{f=1}^{M}\delta_f X_{if}^{-\beta}\right]^{\frac{-1}{\beta}} \qquad (2)$$

where $\delta_f=\alpha_f\Big/\sum_f\alpha_f$ (replasced by α_f when $\sum_f\alpha_f=1$ is used). Clearly the solution is a generalized mean which includes the weighted arithmetic, harmonic and geometric mean.

As may be verified, these ideal functions forms are independent of whether we work with S_i as a function of the shares, $x_{if}=X_{if}\Big/\sum_{i=1}^{N}X_{if}$, or the absolute levels, X_{if} ,(see Maasoumi 1986).

In our numerical investigation, we use $S_i=[\sum_{f=1}^{M}\delta_f X_{if}^{-\beta}]^{\frac{-1}{\beta}}$ for $\beta=-1,0$.

3. MEASURES OF INEQUALITY

The Gini index of inequality is the most well known and widely used measure in empirical studies. The recent influence of developments in the axiomatic approach to measures of inequality and the desire for less ambiguous decompositions of inequality into constituent and population subgroups is focusing attention on a new class of inequality measures known as the generalized entropy (GE) (see Bourguignon 1979; Shorrocks 1980; Maasoumi 1993). The relative inequality in $S = (S_1, S_2, ..., S_N)$ by the GE family defined as follows:

$$I_\gamma(S) = \frac{1}{\gamma(\gamma+1)} \sum_{i=1}^{N} p_i [(\frac{S_i^*}{p_i})^{\gamma+1} - 1], \qquad \gamma \neq 0, -1, \qquad (3a)$$

$$= \sum_{i=1}^{N} S_i^* \log(\frac{S_i^*}{p_i}), \qquad \gamma = 0, \qquad (3b)$$

$$= \sum_{i=1}^{N} p_i \log(\frac{p_i}{S_i^*}), \qquad \gamma = -1. \qquad (3c)$$

where $S_i^* = S_i \Big/ \sum_j S_j$ and $p_i (= \frac{1}{N})$ is the ith unit's population share. I_1 is ordinally equivalent to the Coefficient of Variation and Herfindahl measure, I_0 and I_{-1} are the Theil's first and second measures, respectively. For $\gamma < 0$ the GE measures are ordinally equivalent to the following measures proposed by Atkinson(1970):

$$A_\nu(S) = 1 - \left[\sum_{i=1}^{N} p_i (\frac{S_i^*}{p_i})^{1-\nu} \right]^{\frac{1}{1-\nu}}, \qquad \nu > 0, \ \nu \neq 1,$$

$$= 1 - \exp\left[\sum_i p_i \log(\frac{S_i^*}{p_i}) \right], \qquad \nu = 1.$$

$\nu = -\gamma$ is referred to as the "degree of relative inequality aversion." As noted by Shorrocks(1980) and Atkinson(1970), the GE and Atkinson measures are homogeneous (in a single attribute), symmetric, and consistent with the Lorenz criterion with respect to S_i. Useful decomposability properties of $I_\gamma(\,)$ are explicitly given in Maasoumi(1986) which separate the "between" and "within" group inequalities for any partitioning of the population.

In our numerical investigation, we use both of Theil's measures, $I_0(S)$ and $I_{-1}(S)$, . These two measures are decompose as follows:

$$I_0(S) = \sum_{f=1}^{M} C_f \, I_0(X_f) - D_{-1}^* \qquad \sum_f C_f = 1, \qquad (4)$$

$$I_{-1}(S) = \sum_{f=1}^{M} \delta_f \, I_{-1}(X_f) - D_0^* \ . \qquad (5)$$

where $I_\gamma(X_f)$ is the inequality in the fth variable and D^*_β is the minimum of D_β obtained at $S_i = \sum_f \delta_f X_{if}$ for $\beta = -1$, and at $S_i = \left(\prod_f X_{if}\right)^{\delta_f}$ for $\beta = 0$ and $C_f = \frac{T_f}{\sum_f T_f}$ where $T_f = \sum_i X_{if}$ (see Maasoumi 1986). These decompositions are clearly revealing of the contribution of each welfare variable to multidimensional inequality.

4. NUMERICAL DEMONSTRATION BASED ON IRAN DATA

In this section we report the result of computation on Iran data of householdes income study. There were 12337 household in this study of which 13 ones were not considered in the analysis because of inadequate information. So the sample size decreased to 12324.

Our first attribute is annual nominal income. This was defined to include wages and salaries, business income, and such things as asset income and transfer payments to the household. The second attribute is the number of rooms. In Iran, one of the welfare attributes is the number of rooms which they have. The families who have more welfare have more rooms in comparison with others. The Third attribute is the building area. The richer family has the more area building and the poorer family has less. Our unit of measurement is the "household", and household attributes are adjusted for family size by simply dividing by the number of individuals in the household(for income, the number of rooms and building area). The fourth attribute is home appliance. According to the average price of the home appliances we consider a weight for each of them. Home appliances and their weights are as follows: car(.6751), motor-cycle(.0253), bicycle(.0084), sewing-machine(.0035), radio(.0008), tape recorder(.0051), withe and black TV(.0084), color TV(.0253), freezer(.0253), refrigerator(.0169), oven(.0169), vacuum cleaner(.0118), washing machine(.0169), video(or VCD) player(.0084), computer(.0675) and mobile(.0844). The fifth attribute is home utilities. Home utilities include plumbing, electricity, gas, bath, cooler, telephone, central air condition and kitchen. For cooler and central air condition we assign the weight equal to 0.5 and for other utilities we weightining equal to 1 . If a family has all of them its utility variable equal to 7 and so on if the family use less utility of the package its utility variable would be less than 7.

In table 1, three inequality measures including Theil's measures(equations 3b and 3c) and Gini is reported for all of the attributes. In this table, X1 to X5 are annual nominal income, number of rooms, building area, home appliances and home utilities, respectively. In table 2 and 3, inequality in welfare aggregates, S_{-1} and S_0, computing using $S_i = [\sum_{f=1}^{M} \delta_f X_{if}^{-\beta}]^{\frac{-1}{\beta}}$ for $\beta = 0$ and -1 and decomposition forms according to relations 4 and 5 respectively, reported. In all of our computations we set $\delta_f = \frac{1}{M} = \frac{1}{5}$.

Table 1 : Inequality in welfare attributes

	X1	**X2**	**X3**	**X4**	**X5**
$I_0(X)$	.335214	.190314	.246482	.626885	.030091
$I_{-1}(X)$	.305666	.163385	.240096	.647494	.049749
Gini	.418443	.310278	.373457	.565690	.120608

Table 2 : Inequality in welfare aggregate, S_{-1} and decomposition form

$\sum_{f=1}^{5} C_f I_0(X^f)$	.335212912
D_{-1}^{*}	.000001393
$I_0(S_{-1})$	.335211519

Table 3 : Inequality in welfare aggregate, S_0 and decomposition form

$\sum_{f=1}^{5} \delta_f I_{-1}(X_f)$	.281277931
D_0^{*}	.14744496
$I_{-1}(S_0)$	.133832971

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