

Supplement: Perfect Bayesian Equilibria at Borderline Cost Conditions

At the borders between the cost regimes (low, separating, mixed, and high costs), there exist additional PBEs. Because these PBEs depend on a knife-edge cost condition, they are substantively unimportant except in extremely specific circumstances. They are noted here for the sake of completeness and for their theoretical interest.

The border between separating and low cost: $(c_w \leq 1-2x) \wedge (c_{uw} = 1-x)$. Let us first examine uw type's best response function. If R trusts the signal, $u(s)=1-(1-x) = x$ and $u(ns) = x$. Thus, uw type E is indifferent between sending and not sending the signal. If R does not trust the signal, uw type's best response is not to send the signal. For the w type, if R trusts the signal, sending the signal is the best response. If R does not trust the signal, not sending the signal is w type's best response. R trusts the signal if $\mu_{w|s} \geq \frac{x}{1-x}$. Suppose all w type send the signal and the uw type sends the signal with probability q . Then $\mu_{w|s} = \frac{p_w}{p_w+(1-p_w)q}$. R trusts the signal if $\frac{p_w}{p_w+(1-p_w)q} \geq \frac{x}{1-x}$, or $q \leq \frac{p_w(1-2x)}{(1-p_w)x}$. This result implies that for each (c_w, c_{uw}) with $(c_w \leq 1-2x) \wedge (c_{uw} = 1-x)$ there exists a continuum of PBEs, regardless of p_w , in which R trusts the signal, w type always sends the signal, and the uw type sends the signal with probability $q \leq \frac{p_w(1-2x)}{(1-p_w)x}$. Are these outcomes supportable by an evolutionary stable proportion of types? In the R position, payoffs are the same for both types: $\frac{p_w}{p_w+(1-p_w)q}(1-x) + \frac{(1-p_w)q}{p_w+(1-p_w)q}(0)$. In the E position, the uw type's fitness is x and the w type gets $1-x-c_w$. Thus, if $x = 1-x-c_w$, or $c_w = 1-2x$, the mixed strategy PBE is supported by an evolutionary stable proportion of types for any p_w . If $c_w < 1-2x$, the mixed strategy PBE is

supported by an evolutionary stable $p_w = 1$. If $p_w \geq \frac{x}{1-x}$, then there also exists a Generalized Trust PBE as previously defined (that is not supported by an evolutionary stable proportion of types.)

The border between high cost and mixed cost: $(c_w > 1-2x) \wedge (c_{uw} = 1-x)$. Here, not sending the signal is a dominant action for the w type due to the high signaling costs. Thus, signals are never trusted and the uw type never sends the signal either. The PBE is Generalized Trust if $p_w \geq \frac{x}{1-x}$, and Generalized Distrust if $p_w < \frac{x}{1-x}$.

The border between separating and high cost: $(c_w = 1-2x) \wedge (c_{uw} > 1-x)$. If R trusts the signal, w type is indifferent between sending and not sending the signal. Sending the signal is a dominated action for the uw type. Thus, there exist, for all p_w , a continuum of PBEs in which the w type sends the signal with probability q ($0 < q < 1$) and the R trusts the signal. Can this equilibrium be supported by an evolutionary stable proportion of types? In the R position the two types have the same fitness regardless of q . In the E position, the uw type gets x and the w type gets x as well, regardless of q . Thus, any proportion p_w is evolutionary stable.

The border between low cost and mixed cost: $(c_w = 1-2x) \wedge (c_{uw} < 1-x)$. Here, the uw type wants to send the signal if it is trusted. The w type is indifferent. Suppose the uw type sends the signal with probability 1 and the w type sends the signal with probability q . From previous analysis, we know that $\mu_{w|s} \geq \frac{x}{1-x}$ is the sequential rationality condition for an R player to trust the signal. Here, that implies $\mu_{w|s} = \frac{qp_w}{qp_w + (1-p_w)} \geq \frac{x}{1-x}$, or $q \geq \frac{(1-p_w)x}{p_w(1-2x)}$. This PBE does not exist if $p_w < \frac{x}{1-x}$. In general the PBE requires a larger q as p_w gets smaller and x gets larger. The generalized trust PBE exists if $p_w \geq \frac{x}{1-x}$. The generalized distrust PBE is the only PBE if $p_w < \frac{x}{1-x}$.

Are there any evolutionary stable PBEs in this border? Generalized distrust PBE's are evolutionary stable for any $p_w < \frac{x}{1-x}$. Are there any mixed strategy evolutionary stable PBEs? (We know now that we only need to compare the two types' fitness in the E position.) In the mixed strategy PBE described above, w type gets x and the uw type gets more than x , causing p_w to decline below the threshold $p_w = \frac{x}{1-x}$ needed to support the equilibrium. Thus, in this borderline, the generalized distrust PBE is the only evolutionary stable PBE.

The intersection of all four cost regimes: $(c_w = 1-2x) \wedge (c_{uw} = 1-x)$. Here, when the signal is trusted, both types are indifferent between sending and not sending the signal. Let q_w and q_{uw} be the respective probabilities that w and uw types send the signal. The probability that a signal sender is the w type is $\frac{p_w q_w}{p_w q_w + (1-p_w) q_{uw}}$, which has to be at least $\frac{x}{1-x}$ for R to trust the signal. That is, $q_w \geq \frac{(1-p_w)x}{p_w(1-2x)} q_{uw}$. All PBE's at this intersection are evolutionary stable for any p_w (every type makes the same payoff in any equilibrium.)