MATH 112 Formula Sheet

Basic Trigonometry:

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$
0	0	1	0
$\pi/6$	$1/2$	$\sqrt{3}/2$	$\sqrt{3}/3$
$\pi/4$	$\sqrt{2}/2$	$\sqrt{2}/2$	1
$\pi/3$	$\sqrt{3}/2$	$1/2$	$\sqrt{3}$
$\pi/2$	1	0	undef.

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \sec\theta = \frac{1}{\cos\theta}$$
$$\cot\theta = \frac{\cos\theta}{\sin\theta} \qquad \csc\theta = \frac{1}{\sin\theta}$$

Pythagorean Identities:

1. $\sin^2 x + \cos^2 x = 1$
2. $\tan^2 x + 1 = \sec^2 x$
3. $1 + \cot^2 x = \csc^2 x$

Half Angle Identities:

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x) \qquad \cos^2 x = \frac{1}{2}(1 + \cos 2x)$$

Other Identities:

1. $\sin A \cos B = \frac{1}{2}[\sin(A-B) + \sin(A+B)]$
2. $\sin A \sin B = \frac{1}{2}[\cos(A-B) - \cos(A+B)]$
3. $\cos A \cos B = \frac{1}{2}[\cos(A-B) + \cos(A+B)]$

Inverse Trig Functions:

$f(x)$	Domain	Range	Derivative
$\sin^{-1} x$	[-1,1]	$[-\pi/2, \pi/2]$	$f'(x) = \frac{1}{\sqrt{1-x^2}}$
$\cos^{-1} x$	[-1,1]	$[0, \pi]$	$f'(x) = \frac{-1}{\sqrt{1-x^2}}$
$\tan^{-1} x$	$(-\infty, \infty)$	$(-\pi/2, \pi/2)$	$f'(x) = \frac{1}{1+x^2}$

Integration Formulas:

$$\int \tan x \, dx = \ln|\sec x| + C \qquad \int \sec x \, dx = \ln|\sec x + \tan x| + C$$

Trigonometric Substitution:

Form	Substitution	Identity
$\sqrt{a^2 - x^2}$	$x = a \sin\theta$	$1 - \sin^2\theta = \cos^2\theta$
$\sqrt{a^2 + x^2}$	$x = a \tan\theta$	$1 + \tan^2\theta = \sec^2\theta$
$\sqrt{x^2 - a^2}$	$x = a \sec\theta$	$\sec^2\theta - 1 = \tan^2\theta$

Error Bounds:
Suppose $|f''(x)| \leq K$ for $a \leq x \leq b$. If E_T and E_M are the errors in the Trapezoid and Midpoint Rules, then

$$|E_T| \leq \frac{K(b-a)^3}{12n^2} \quad \text{and} \quad |E_M| \leq \frac{K(b-a)^3}{24n^2}$$

Arc Length of a Parametrically Defined Curve
$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

Surface Area of a Surface of Revolution
$S = \int_a^b 2\pi y \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

Arc Length of a Polar Curve
$L = \int_a^b \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$