Section 8.7

We sometimes wish to approximate the value of a definite integral when we are unable to evaluate it exactly. We can estimate the area bound under a curve by covering the area with simple geometric shapes, finding their areas, and adding them together. We discuss three different techniques to accomplish this.

Suppose f is an integrable function on a closed interval $[a, b]$ which has been partitioned $\{a = x_0, x_1, x_2, \ldots, x_n = b\}$ into n equal subintervals of length $\Delta x = \frac{b-a}{n}$. We may then approximate $\int_a^b f(x)\,dx$ using one of the following rules:

1. Midpoint Rule: Let $\overline{x}_i$ be the midpoint of the ith subinterval.
   That is, $\overline{x}_i = \frac{1}{2}[x_{i-1} + x_i]$. Then,
   $\int_a^b f(x)\,dx \approx \Delta x[f(\overline{x}_0) + f(\overline{x}_1) + f(\overline{x}_2) + \cdots + f(\overline{x}_{n-1}) + f(\overline{x}_n)]$
2. Trapezoidal Rule:
   $\int_a^b f(x)\,dx \approx \frac{\Delta x}{2}[f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-1}) + f(x_n)]$
3. Simpson's Rule: To apply Simpson's Rule, n must be even.
   $\int_a^b f(x)\,dx \approx \frac{\Delta x}{3}[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) \cdots + 4f(x_{n-1}) + f(x_n)]$

The maximum error for each of the three methods can be determined by using the following error bounds:

Suppose $|f''(x)| \leq K$ for $a \leq x \leq b$. If E_T and E_M are the errors in the Trapezoid and Midpoint Rules, then

$$|E_T| \leq \frac{K(b-a)^3}{12n^2} \quad \text{and} \quad |E_M| \leq \frac{K(b-a)^3}{24n^2}$$

Suppose $|f^{(4)}(x)| \leq K$ for $a \leq x \leq b$. If E_S is the error in Simpson's Rule, then

$$|E_S| \leq \frac{K(b-a)^5}{180n^4}$$

Section 8.8