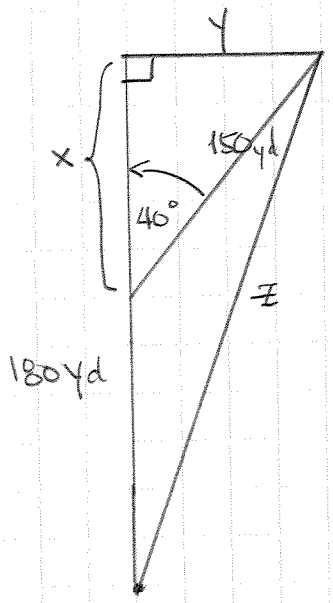
Math 115 · Fall 2009

Sample Solutions

#63 p. 48



(a) Find x. (requires calculator with angles in degrees)

$$\cos 40^\circ = \frac{x}{150}$$

$$\Rightarrow x = 150 \cos 40^\circ \doteq 150 \cdot (.7660)$$

$$\doteq 114.9067 \text{ yd.}$$

(b) Find y

$$\tan 40^\circ = \frac{y}{x} \Rightarrow y = x \tan 40^\circ$$

$$\doteq (114.9067)(.8391)$$

$$\doteq 96.4182 \text{ yd}$$

(c) Find z

$$z^2 = (x+180)^2 + y^2 \doteq (114.9067 + 180)^2 + (96.4182)^2$$

$$\doteq 96266.431$$

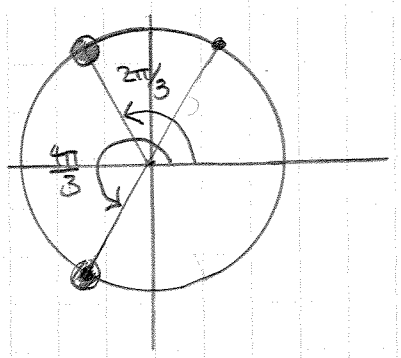
$$\therefore z \doteq \sqrt{96266.431} \doteq 310.2683 \text{ yd.}$$

#29 p. 58 Find all solutions t of $\cos\left(3t + \frac{\pi}{4}\right) = -\frac{1}{2}$.

$\cos\theta = -\frac{1}{2}$. For θ positive,

$\theta = \frac{2\pi}{3}, \frac{2\pi}{3} + 2\pi, \frac{2\pi}{3} + 4\pi, \ldots$

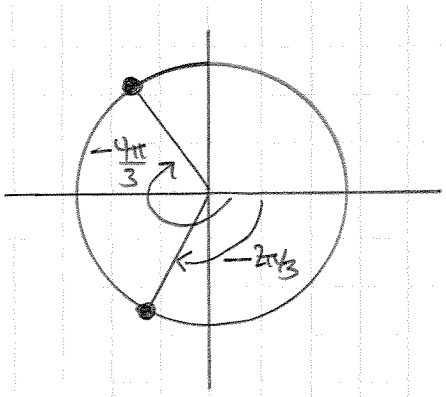
and $= \frac{4\pi}{3}, \frac{4\pi}{3} + 2\pi, \frac{4\pi}{3} + 4\pi, \ldots$



The negative solutions are

$\theta = -2\pi/3, -2\pi/3 - 2\pi, -2\pi/3 - 4\pi, \ldots$

and $= -4\pi/3, -4\pi/3 - 2\pi, -4\pi/3 - 4\pi, \ldots$



Let's continue with θ positive

(i) $\theta = 2\pi/3 + 2n\pi$ $\quad n = 0, 1, 2, \ldots$

$3t + \pi/4 = 2\pi/3 + 2n\pi$

$\Rightarrow \quad 3t = 5\pi/12 + 2n\pi$

$\Rightarrow \quad t = 5\pi/36 + \frac{2}{3}n\pi \quad ; \quad n = 0, 1, 2, \ldots$

(ii) $\theta = 4\pi/3 + 2n\pi$ $\quad n = 0, 1, 2, \ldots$

$3t + \pi/4 = 4\pi/3 + 2n\pi$

$3t = 13\pi/12 + 2n\pi$

$t = 13\pi/36 + \frac{2}{3}n\pi \quad ; \quad n = 0, 1, 2, \ldots$

#33 p. 58 Find all solutions t of $2\sin^2 t - 5\sin t - 3 = 0$

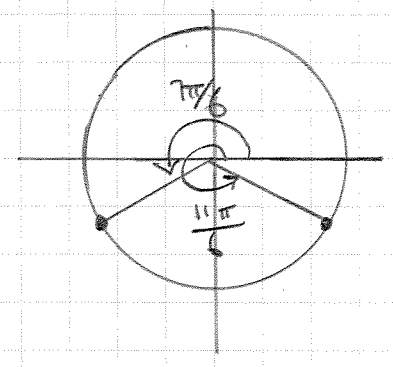
let $y = \sin t$. Solve $2y^2 - 5y - 3 = 0$

Factor : $(2y+1)(y-3) = 0 \iff 2y+1 = 0$ or $y - 3 = 0$

But $y = \sin t$ so $y - 3 = 0 \iff \sin t = 3$, which has no solution

So we need only solve $2 \cdot \sin t + 1 = 0 \iff \sin t = -1/2$

Let's restrict to positive t again.



$t = \frac{7\pi}{6} + 2n\pi$; $n = 0, 1, 2, \cdots$

and $\frac{11\pi}{6} + 2n\pi$; $n = 0, 1, 2, \cdots$

#53, p. 59

Let $V(t)$ be the volume of air in lungs at time t sec in mL

We assume $V(t) = a\cos bt + k$ (since we know $V(t)$ is given by a sinusoidal function and $V(t)$ is maximized at $t = 0$, so we use $a\cos bt + k$ rather than $a \sin bt + k$.)

Find a, b, k, assuming

period = 5 sec, average (midline) = 2500 mL,

tidal volume = 500 mL.

$k = 2500$

$500 = \max - \min = a - (-a) = 2a \quad \therefore a = 250$

$5 = \text{period} = 2\pi/b \quad \therefore b = 2\pi/5$

Thus, $V(t) = 250 \cdot \cos\left(\frac{2\pi}{5}t\right) + 2500$ [mL].