

Dynamic Multi-Activity Contests

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Abstract

In many contests, players can influence the outcome through efforts in multiple activities, several of which can be chosen before others. In this paper, we develop a model of dynamic multi-activity contests. Players simultaneously choose efforts in long-run activities, observe each other's efforts in these activities, and then simultaneously choose efforts in short-run activities. A player's long-run and short-run efforts complement each other in determining the player's probability of winning. We compare the outcomes of this two-stage model to those of the corresponding model in which players do not observe each other's first-stage efforts before the second stage and thus effectively choose efforts in all activities simultaneously. Interestingly, effort expenditures are always lower in the sequential multi-activity contest than in the simultaneous multi-activity contest. The implications of this result for the organization of military, litigation, innovation, academic, and sporting contests are highlighted.

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1. Introduction

In many contests, players can influence their chances of winning through efforts in multiple activities, several of which can be chosen initially or in the long run and several of which can be chosen later or in the short run. For example, in military contests, enemies can choose how much to spend on weapons-development programs and then choose expenditures in actual fighting. In legal contests, litigants can choose efforts in collecting initial evidence for trial and then choose efforts at the trial stage. In R&D contests, firms can choose expenditures on prototype-development or quality-improvement and then choose expenditures on commercialization or advertising. In academic contests, parents can choose how much to invest in their children's early education and then choose how much to invest in education later in life. In sporting contests, athletes can choose efforts to train for a match and then choose efforts in the actual match. In general, a participant's long-run and short-run efforts interact to determine his or her chances of winning the contest.

To examine the implications of the dynamic multidimensional structure of many contests, we develop a model of contests in which players have multiple activities by which to influence the outcome and can commit to a subset of these activities (the long-run activities) before making choices for the remaining activities (the short-run activities). In the model, players simultaneously choose efforts in the long-run activities, observe each other's long-run efforts, and then simultaneously choose efforts in the short-run activities. Each player's probability of winning is assumed to be a function of both long-run and short-run efforts. Specifically, we employ the Cobb-Douglas type contest success function, which is axiomatized by Arbatskaya and Mialon (2010) for multi-activity contests.

This specification assumes complementarity between a player's efforts at different stages, which is realistic in many contexts. The technology of human skill formation is fundamen-

tally characterized by dynamic complementarity: skills produced at one stage tend to raise the productivity of investment at subsequent stages. Cunha et al. (2010) identify dynamic complementarity in the development of both cognitive and non-cognitive skills, and Cunha and Heckman (2007) organize the large body of evidence on such complementarity in skill formation from psychology, education, and neuroscience. In the academic context, educational efforts in early childhood and later in life do not simply add up, but compound. Efforts across time may compound through other processes as well. For example, in the R&D context, product-quality improvements and advertising may be complementary because of the greater benefit that firms with high-quality products enjoy from providing direct product-quality information through advertisements (Bagwell, 2007, and Caves and Greene, 1996).

We analyze the outcomes of the two-stage Cobb-Douglas multi-activity model of contests and then compare these outcomes to those of the corresponding model in which players do not observe each other's first stage efforts before choosing their second stage efforts and thus effectively choose efforts in all activities simultaneously. We ask whether effort expenditures in each activity and overall are higher or lower in the sequential (two-stage) multi-activity contest than in the corresponding simultaneous (one-stage) multi-activity contest.

Several interesting results arise. First, the overall stronger player's long-run efforts are higher relative to the overall weaker player's long-run efforts in the two-stage multi-activity contest than in the one-stage multi-activity contest. Second, both players' short-run efforts are lower in the sequential contest. Intuitively, in the sequential contest, the players strategically choose their long-run efforts to unbalance short-run competition, with the result that short-run efforts are lower. Third, total effort expenditures and rent dissipation are lower in the two-stage contest because the two-stage contest is always effectively more unbalanced than the one-stage contest.

These results have potentially important implications for the optimal design of contests. In dynamic multi-activity contests, if long-run efforts are private information and the social objective is to minimize effort costs, it might be beneficial to mandate that players disclose their long-run efforts before making their short-run choices since doing so would lower overall effort costs. For example, in military contests, mandatory disclosure of weapons-development efforts might lower overall fighting costs. In legal battles, mandatory disclosure of preliminary evidence collection and lawyer compensation might lower overall legal expenditures.

If the objective is to promote effort expenditures, it might be beneficial to prevent players from observing each other's long-run efforts. For example, in R&D contests, protecting the secrecy of prototype-development and quality-improvement efforts might increase the total of expenditures on innovation and commercialization. In academic contests, securing the confidentiality of primary education records might increase the aggregate expenditures on primary and higher education. In sporting contests, encouraging athletes to train in secret might increase the sum of expenditures on training and match play.

In Section 2, we develop the two-stage contest model with two activities, solve for the equilibrium of the short-run stage with efforts in the long-run activity fixed, and derive the equilibrium of the two-stage contest. Section 3 compares the outcomes of the two-stage model to those of the corresponding one-stage model. Section 4 shows that the analysis extends to any number of short-run and long-run activities. Section 5 relates the paper to existing literature. Section 6 summarizes and discusses avenues for future work.

2. Two-Stage Two-Activity Contest

Consider a dynamic all-pay contest between two players, $i = 1, 2$, who are competing for a prize that each values at $v > 0$ by choosing efforts in two activities, x_i and y_i , where efforts in

activity x are chosen in the long-run while efforts in activity y are chosen in the short run.²

Players first simultaneously choose their efforts in the long-run activity, x_1 and x_2 (stage 1). Then, after observing each other's efforts in the long-run activity, they simultaneously choose their efforts in the short-run activity, y_1 and y_2 (stage 2). We assume that players have linear costs of effort, with positive marginal costs c_1 and c_2 in long-run efforts and d_1 and d_2 in short-run efforts.

Player i 's payoff in the two-stage, two-activity contest is then:

$$\Pi_i(x_1, x_2, y_1, y_2) = p_i(x_1, x_2, y_1, y_2) v - c_i x_i - d_i y_i, \quad (1)$$

where $p_i = p_i(x_1, x_2, y_1, y_2)$ is player i 's *contest success function*, or player i 's winning probability as a function of both players' efforts in the two activities. We assume that when at least one of the players makes a positive effort in the two activities, player i 's contest success function has the logistic form, $p_i(x_1, x_2, y_1, y_2) = \frac{f(x_1, y_1)}{f(x_1, y_1) + f(x_2, y_2)}$, and otherwise, it is equal to zero. The influence production function f is assumed to be of Cobb-Douglas type $f(x_i, y_i) = x_i^{\alpha_x} y_i^{\alpha_y}$, where $\alpha_x > 0$ and $\alpha_y > 0$ measure the discriminatory power (or decisiveness) of the contest in long-run and short-run efforts.³

The overall discriminatory power of the contest is defined as $\alpha \equiv \alpha_x + \alpha_y$, and the overall relative strength of player 2 in the contest is defined as $\theta \equiv \left(\frac{c_1}{c_2}\right)^{\alpha_x} \left(\frac{d_1}{d_2}\right)^{\alpha_y}$. If $\theta = 1$, then the contest is symmetric overall; if $\theta > 1$, player 2 is stronger overall than player 1; and if $\theta < 1$, player 1 is stronger overall. The proximity between θ and 1 captures the extent to which the contest is balanced.

² We present our analysis for the case of two activities to simplify the exposition. We prove in Section 4 that the analysis and all of the results in the paper extend in a straightforward manner to the general case of any number of long-run and short-run activities.

³ For an axiomatization of this contest success function for multi-activity contests, see Arbatskaya and Mialon (2010), where it is shown that a contest success function has to be of this form to satisfy such properties as independence from irrelevant alternatives and invariance to units of measurement for each activity as well as several more basic properties of well-behaving contest success functions.

The solution to the two-stage contest is found by backward induction. We start by analyzing the stage-2 (short-run) contest in which the long-run efforts are fixed at levels x_1 and x_2 , and players choose short-run efforts y_1 and y_2 . The contest with some activities fixed is interesting in its own right, as a multi-activity contest with binding asymmetric caps on some activities. Then, we solve for the pure-strategy subgame perfect equilibrium $(x_1^*, x_2^*, y_1^*, y_2^*)$ in the two-stage contest, and we compare this equilibrium to the pure-strategy Nash equilibrium $(x_1^{*1}, x_2^{*1}, y_1^{*1}, y_2^{*1})$ obtained in the one-stage contest where all efforts are chosen simultaneously. Our main objective is to establish that the total cost of effort $C^* \equiv c_1 x_1^* + c_2 x_2^* + d_1 y_1^* + d_2 y_2^*$ and rent dissipation $D^* \equiv C^*/v$ are lower in a two-stage contest than in the corresponding one-stage contest.

We already know the equilibrium results for the one-stage two-activity contest from Proposition 2 in Arbatskaya and Mialon (2010). In our current notation, the main results are the following: there exists a unique Nash equilibrium to the one-stage two-activity contest if $\alpha \in (0, 1]$ or $\alpha \in (1, 2]$ and $\theta \in [(\alpha - 1), (\alpha - 1)^{-1}]$, i.e., the overall discriminatory power of the contest is not too high and the contest is not too unbalanced. In the equilibrium, player i 's equilibrium efforts are $x_i^{*1} = \frac{\alpha_x v \theta}{c_i} (1 + \theta)^{-2}$ and $y_i^{*1} = \frac{\alpha_y v \theta}{d_i} (1 + \theta)^{-2}$, and rent dissipation is $D^{*1} = 2\alpha\theta (1 + \theta)^{-2}$.

2.1 Solution to Stage 2 of Two-Stage Two-Activity Contest

At stage 2, the long-run activities are fixed at levels $x_1 > 0$ and $x_2 > 0$, and we look for an interior equilibrium $(y_1^*(x_1, x_2), y_2^*(x_1, x_2))$. Player i 's payoff is

$$\Pi_i(x_1, x_2, y_1, y_2) = \left(\frac{x_i^{\alpha_x} y_i^{\alpha_y}}{x_1^{\alpha_x} y_1^{\alpha_y} + x_2^{\alpha_x} y_2^{\alpha_y}} \right) v - c_i x_i - d_i y_i \quad (2)$$

when $y_1 > 0$ or $y_2 > 0$, and it is $-c_i x_i - d_i y_i$ otherwise. We can interpret $x_1^{\alpha_x}$ and $x_2^{\alpha_x}$ as players' ability achieved through investments in the long-run activity. In the pure-strategy

Nash equilibrium to the stage-2 subgame, player i chooses his short-run effort y_i to maximize his payoff given the equilibrium effort level chosen by the opposing player.

In a stage-2 contest with the efforts in the long-run activity fixed at levels x_1 and x_2 , the effective relative strength of player 2 is defined as $\bar{\theta} \equiv \left(\frac{x_2}{x_1}\right)^{\alpha_x} \left(\frac{d_1}{d_2}\right)^{\alpha_y}$. The value of $\bar{\theta}$ is the product of the relative ability of player 2 due to investments in the long-run activity, $\left(\frac{x_2}{x_1}\right)^{\alpha_x}$, and player 2's relative strength in the short-run activity, $\left(\frac{d_1}{d_2}\right)^{\alpha_y}$. In the short run, players are effectively symmetric if $\bar{\theta} = 1$; player 2 is effectively stronger than player 1 if $\bar{\theta} > 1$; and player 1 is effectively stronger if $\bar{\theta} < 1$.

Proposition 1 gives the conditions under which the Nash equilibrium in the short-run contest exists and is unique and gives the equilibrium efforts and total cost of effort.⁴

Proposition 1. *There exists a unique Nash equilibrium in the stage-2 contest if $\alpha_y \in (0, 1]$ or $\alpha_y \in (1, 2]$ and $\bar{\theta} \in [(\alpha_y - 1), (\alpha_y - 1)^{-1}]$. In the equilibrium, player i 's effort and the total cost of effort in the short-run activity are $y_i^* = \frac{\alpha_y v \bar{\theta}}{d_i} (1 + \bar{\theta})^{-2}$ and $C_y^* = d_1 y_1^* + d_2 y_2^* = 2\alpha_y v \bar{\theta} (1 + \bar{\theta})^{-2}$.*

The equilibrium cost of effort in the short-run activity is the same for the two players, $d_1 y_1^* = d_2 y_2^* = \alpha_y v \bar{\theta} (1 + \bar{\theta})^{-2}$, and it depends on the effective relative strength of player 2 in the short-run, $\bar{\theta} = \left(\frac{x_2}{x_1}\right)^{\alpha_x} \left(\frac{d_1}{d_2}\right)^{\alpha_y}$. Proposition 1 implies that the effect of a player's long-run investment on the effort in the short-run activity is ambiguous, with $\text{sign}\left(\frac{\partial y_i^*}{\partial x_1}\right) = \text{sign}(\bar{\theta} - 1)$ and $\text{sign}\left(\frac{\partial y_i^*}{\partial x_2}\right) = \text{sign}(1 - \bar{\theta})$. Thus, if $\bar{\theta} < 1$, i.e., player 1 is effectively stronger than player 2 at stage 2, then $\frac{\partial y_i^*}{\partial x_1} < 0$; if $\bar{\theta} > 1$, i.e., player 2 is effectively stronger, then $\frac{\partial y_i^*}{\partial x_1} > 0$; and if $\bar{\theta} = 1$, then $\frac{\partial y_i^*}{\partial x_1} = 0$. Hence, we have the following result:

Corollary 1. *A marginal increase in long-run effort by a player decreases (increases) both*

⁴ Proofs of Propositions 1-3 and Corollary 1 are provided in Appendix A.

players' short-run efforts if the player is effectively stronger (weaker), and it has no effect when players are effectively symmetric in the short-run.

Importantly, additional long-run investments by player i discourage subsequent short-run efforts whenever player i is the stronger player. In the next section, we will show that players strategically choose their long-run efforts to unbalance the contest and reduce short-run rent dissipation.

2.2 Solution to Two-Stage Two-Activity Contest

Consider players' stage-1 choice of long-run efforts x_1 and x_2 , given their anticipation that the contest at stage 2 will result in the equilibrium of Proposition 1. We are looking for an interior subgame perfect Nash equilibrium, with $x_1^* > 0$ and $x_2^* > 0$. At stage 1, player i 's reduced payoff when $x_1 > 0$ or $x_2 > 0$ is

$$\tilde{\Pi}_i(x_1, x_2) = \tilde{p}_i(x_1, x_2)v - c_i x_i - d_i y_i^*(x_1, x_2), \quad (3)$$

where $\tilde{p}_i(x_1, x_2) = p_i(x_1, x_2, y_1^*(x_1, x_2), y_2^*(x_1, x_2))$.

From Proposition 1, the ratio of players' equilibrium efforts in the short-run activity is equal to the inverse of their costs of effort in this activity, $\frac{y_2^*}{y_1^*} = \frac{d_1}{d_2}$, and it is independent of efforts x_1 and x_2 in the long-run activity. Hence, player i 's reduced winning probability is

$$\tilde{p}_i = \left(1 + \left(\frac{x_j}{x_i} \right)^{\alpha_x} \left(\frac{d_i}{d_j} \right)^{\alpha_y} \right)^{-1}. \quad (4)$$

where $i \neq j = 1, 2$. The first-order condition for player i in the long-run effort is

$$\frac{\partial \tilde{\Pi}_i(x_1, x_2)}{\partial x_i} = \left(\frac{\partial \tilde{p}_i}{\partial x_i} \right) v - c_i - d_i \left(\frac{\partial y_i^*}{\partial x_i} \right) = 0. \quad (5)$$

In contrast, in the one-stage contest where players choose efforts in all activities simultaneously, the first-order condition for player i in the long-run activity is

$$\frac{\partial \Pi_i(x_1, x_2, y_1, y_2)}{\partial x_i} = \frac{\partial p_i}{\partial x_i} v - c_i = 0. \quad (6)$$

At the equilibrium levels of short-run efforts, the first term in (5) is the same as the first term in (6) because in both the one-stage and two-stage contests, the ratio of equilibrium efforts with the short-run activity is equal to the inverse ratio of costs. The last term in (5), which does not appear in (6), is the strategic effect of precommitment through long-run efforts on short-run costs. Intuitively, players may want to distort their efforts in the long-run activity at stage 1 in order to further unbalance the short-run competition at stage 2.

Proposition 2 below will indeed show that the overall stronger player's long-run effort is higher relative to the overall weaker player's long-run effort in the two-stage contest, with the result that the two-stage contest is effectively more unbalanced. Proposition 2 also provides the conditions under which the subgame perfect equilibrium exists and is unique in the two-stage contest and characterizes the equilibrium rent dissipation.

Proposition 2. *There exists a unique, interior subgame perfect Nash equilibrium to the two-stage two-activity contest if $\alpha \leq 1$ or if $\alpha \in (1, 2]$, $\alpha_x \leq 1$, $\alpha_y \leq 1$, and $\theta \in [(\alpha - 1)^{1-\alpha_x}, (\alpha - 1)^{\alpha_x-1}]$. The equilibrium level of rent dissipation is $D^* = 2\alpha\theta^* (1 + \theta^*)^{-2}$, where $\theta^* \equiv \left(\frac{x_2^*}{x_1^*}\right)^{\alpha_x} \left(\frac{d_1}{d_2}\right)^{\alpha_y}$ satisfies*

$$\left(\frac{\theta^*}{\theta}\right)^{1/\alpha_x} = \frac{1 - \alpha_y + (1 + \alpha_y)\theta^*}{1 + \alpha_y + (1 - \alpha_y)\theta^*}. \quad (7)$$

For $\theta \neq 1$, the overall stronger player's long-run effort is higher relative to the overall weaker player's long-run effort in the two-stage contest than in the one-stage contest, and the two-stage contest is always effectively more unbalanced than the one-stage contest. For $\theta = 1$, the one-stage and two-stage contests are equally effectively balanced.

To show that the two-stage contest is effectively more unbalanced than the one-stage contest, we compare the overall relative strength of player 2 in the one-stage contest, $\theta = \left(\frac{c_1}{c_2}\right)^{\alpha_x} \left(\frac{d_1}{d_2}\right)^{\alpha_y}$, to the equilibrium effective relative strength of player 2 achieved in the two-

stage contest, $\theta^* \equiv \left(\frac{x_2^*}{x_1^*}\right)^{\alpha_x} \left(\frac{d_1}{d_2}\right)^{\alpha_y}$. The two-stage contest is effectively more unbalanced if θ^* is farther away from one than θ . According to Proposition 2, if player 1 is stronger overall, $\theta < 1$, then player 1's long-run effort is relatively higher in the two-stage contest than in the one-stage contest, $\frac{x_2^*}{x_1^*} < \frac{x_2^{*1}}{x_1^{*1}} = \frac{c_1}{c_2}$, and therefore $\theta^* = \left(\frac{x_2^*}{x_1^*}\right)^{\alpha_x} \left(\frac{d_1}{d_2}\right)^{\alpha_y} < \theta < 1$. Similarly, if player 1 is weaker overall, $\theta > 1$, then $\frac{x_2^*}{x_1^*} > \frac{x_2^{*1}}{x_1^{*1}} = \frac{c_1}{c_2}$ and $\theta^* > \theta > 1$.

If players are symmetric overall, $\theta = 1$, then there is no strategic incentive to distort long-run efforts and thus $\frac{x_2^*}{x_1^*} = \frac{x_2^{*1}}{x_1^{*1}} = \frac{c_1}{c_2}$ and $\theta^* = \theta = 1$. The same result obtains in the limit as $\alpha_y \rightarrow 0$. In this case, the stage-2 activities are not decisive, and therefore players again do not have incentive to distort their stage-1 efforts in an attempt to relax stage-2 competition.

3. Two-Stage v. One-Stage Two-Activity Contests

We now compare the equilibrium total cost of effort and rent dissipation in the two-stage and one-stage contests:

Proposition 3. *The equilibrium rent dissipation and total cost of effort are lower in the two-stage two-activity contest than in the one-stage two-activity contest when $\theta \neq 1$, and they are equal in the two-stage and one-stage two-activity contests when $\theta = 1$.*

Intuitively, players choose long-run efforts strategically to effectively unbalance the contest: the overall stronger player spends more on the long-run activity relative to the overall weaker player in the two-stage contest than in the one-stage contest. This implies that the sequential contest is effectively more unbalanced and the equilibrium efforts in the short-run activity are lower in the two-stage contest than in the one-stage contest. Overall, the two-stage contest results in lower levels of rent dissipation than the one-stage contest. Since the total equilibrium payoff is $\Pi^* \equiv \Pi_1^* + \Pi_2^* = v(1 - D^*)$, the result implies that a contest designer can increase the players' total payoff by organizing the contest in two stages.

4. Two-Stage K -Activity Contests

All the analysis in this paper extends to the general case of any number of long-run and short-run activities. We prove this in Appendix B. Intuitively, the results generalize to the case of multidimensional efforts at each stage because we can reduce a two-stage K -activity contest to a two-stage two-activity contest. To do this, we define x_i and y_i to be player i 's aggregate efforts in stages 1 and 2 and argue that a player will choose the combination of components that generate given short-run and long-run aggregate efforts in a cost-minimizing way. Then, we use the statements of Propositions 1-3 for two-stage two-activity contests to obtain the same results for the corresponding two-stage K -activity contests.

5. Related Literature

Our paper extends the contest literature by analyzing dynamic multi-activity contests. The literature has focused mostly on contests in which players can influence the outcome through only a single activity, with a few exceptions. Konrad (2000), Chen (2003), Kräkel (2005), and Amegashie and Runkel (2007) develop models in which players simultaneously choose effort to improve their performance and effort to sabotage their opponent's performance. In our model, players choose efforts in multiple activities to improve their own performance. Epstein and Hefeker (2003) develop a contest model in which players simultaneously choose efforts in two activities to influence the outcome, and Arbatskaya and Mialon (2010) analyze the effects of restricting activities in multi-activity contests where all the activities are chosen simultaneously. We consider dynamic multi-activity contests, allowing for the possibility that players choose some activities before others.

Several papers model sequential-move contests, but only in the unidimensional case. In these models, a leader and a follower in turn choose efforts in one activity. Dixit (1987,

1999) models the strategic incentives of the leader to precommit to higher effort. With an exogenous order of moves, he shows that if the weaker (stronger) of the two players is the leader, he undercommits (overcommits) relative to the simultaneous-move case. This result is similar in flavor to our result that the stronger player's long-run effort is higher relative to the weaker player's long-run effort in the two-stage than in the one-stage multi-activity contest. Endogenizing the timing of moves, Baik and Shogren (1992) show that the weaker player emerges as the leader and thus rent dissipation is lower in the sequential-move contest. Since these models assume unidimensional effort, they do not address the issue of interaction between long-run and short-run efforts.

Soubeyran (2009) considers a sequential model in which the weaker player allocates a fixed budget between attacking and defensive efforts and then the stronger player does the same, finding that the weaker player may attack less than the stronger player. In the author's model, one player chooses between two efforts in the first stage and the other player chooses between the same two efforts in the second stage. In our model, both players simultaneously choose between several efforts in the first stage and then simultaneously choose between several other efforts in the second stage.

Another related strand of the literature models elimination tournaments and multi-battle contests (Sheremeta, 2010; Konrad and Kovenock, 2009; Fu and Lu, 2010; Clark and Konrad, 2007; Harbaugh and Klumpp, 2006; Amegashie, 2004; and Gradstein and Konrad, 1999). In these models, players engage in a sequence of contests and receive a payoff based on the outcomes of each. A few papers analyze multi-battle contests with accumulation of efforts over time (Schmitt et al., 2004; Lee, 2003; Baik and Lee, 2000). In the settings that are examined, efforts are additive and players face a trade-off between expending efforts earlier or later. Early efforts depreciate but they help players win later contests. Unlike in these

models, efforts do not simply add up, but compound, over time in our model.

Two papers closely related to ours analyze models of contests with investment. Kräkel (2004) develops a model with dynamic complementarity between R&D investments and managerial efforts to raise market shares. The contest success function that he employs is similar to ours, but his focus is on the effects of delegation and R&D spillovers. Münster (2007) develops a model of investment in perfectly discriminating contests or all-pay auctions. In the first stage, two players choose investments to reduce their costs of bidding in the auction, and in the second stage, they choose their bids. The game with investment has equilibria in which rent dissipation is not complete, whereas rent dissipation is always complete in the game with exogenous bid costs. This result is similar in flavor to our result that rent dissipation is lower in the two-stage than in the one-stage multi-activity contest. However, our model is different in that we focus on imperfectly discriminating contests.

Our paper can also be related to oligopoly models in which firms first compete in R&D investment and then engage in price or quantity competition. Qiu (1997) examines two-stage models in which symmetric firms (i.e., firms with the same R&D cost function) simultaneously choose R&D effort in the first stage and simultaneously choose prices or quantities in the second stage, finding that quantity competition induces more R&D effort than price competition. Whereas the author compares two different sequential-move models with multiple activities, we compare the simultaneous and sequential-move cases of one class of models with multiple activities.

Although we do not focus on production market competition, our model may have implications for this type of competition as well. Applied to this context, our model would imply that, if expenditures on R&D are transparent, the firm with a stronger overall position in a market might strategically increase its R&D effort relative to its rival to reduce subsequent

product market competition. Policies that protect the secrecy of R&D efforts might then increase the short-run competition.

6. Conclusion

We have developed a model of dynamic multi-activity contests, in which a contestant's long-run and short-run efforts complement each other. We have shown that the total cost of effort and rent dissipation are unambiguously lower in the two-stage multi-activity contest in which contestants can observe each other's long-run efforts before choosing their short-run efforts than in the one-stage contest in which they must choose all their efforts simultaneously.

It would be interesting to test this prediction of our model. To do so, one could build on the experimental design that Sheremeta (2010) developed to test the multi-stage elimination contest model analyzed by Gradstein and Konrad (1999). A prediction of this model is that total effort is the same in the multi-stage case than in the corresponding one-stage case when the discriminatory power of the elimination contest is equal to one. Contrary to the theory, however, Sheremeta finds that total effort is higher in the multi-stage case.

It may also be interesting to extend our model to the case where early efforts undermine rather than complement later efforts. This may allow for an exploration of strategic motives for self-sabotage. For example, people might choose how much time to spend partying or drinking when young and choose effort to become employed when older. Earlier partying undermines later employment effort. One of the implications of the model for the case when people have the same direct benefit of partying might be that people with higher employment costs increase the extent of their partying relative to others if the extent of earlier partying is observable (e.g., through social networking sites or academic transcripts). That is, they rationally sabotage their own chances of employment to reduce total effort expenditures.

Appendix A: Proofs

Proof of Proposition 1. With the substitution $z_i \equiv x_i^{\alpha_x} y_i^{\alpha_y}$, the payoff of player i in (2) gross of fixed long-run costs can be written as $v z_i / (z_1 + z_2) - d_i x_i^{-\alpha_x / \alpha_y} z_i^{1/\alpha_y}$. From the first-order conditions with respect to z_i , $\frac{z_j}{(z_1 + z_2)^2} v = \frac{d_i}{\alpha_y} x_i^{-\alpha_x / \alpha_y} z_i^{1/\alpha_y - 1}$ for $i \neq j = 1, 2$, it follows that $\left(\frac{z_2}{z_1}\right) = \left(\frac{x_2}{x_1}\right)^{\alpha_x} \left(\frac{d_1}{d_2}\right)^{\alpha_y} = \bar{\theta}$ and therefore, in the equilibrium, $d_i x_i^{-\alpha_x / \alpha_y} z_i^{1/\alpha_y} = \alpha_y v \bar{\theta} (1 + \bar{\theta})^{-2}$. By checking non-negativity of equilibrium payoffs and the second-order sufficient conditions, we find that the interior equilibrium exists if $\alpha_y \in (0, 1]$ or $\alpha_y \in (1, 2]$ and $\bar{\theta} \in [(\alpha_y - 1), (\alpha_y - 1)^{-1}]$. Hence, we conclude that under the same set of parameters there exists a unique pure strategy Nash equilibrium in the stage-2 subgame with the long-run efforts fixed at levels x_1 and x_2 . The equilibrium short-run effort of player i is $y_i^* = x_i^{-\alpha_x / \alpha_y} z_i^{1/\alpha_y} = \frac{\alpha_y v \bar{\theta}}{d_i} (1 + \bar{\theta})^{-2}$, and the total cost of effort in the short-run activity is $C_y^* = d_1 y_1^* + d_2 y_2^* = 2\alpha_y v \bar{\theta} (1 + \bar{\theta})^{-2}$. **Q.E.D.**

Proof of Corollary 1. From Proposition 1, player i 's equilibrium choice of short-run effort is $y_i^* = \frac{\alpha_y v}{d_i} f(\bar{\theta})$, where $f(\bar{\theta}) = \bar{\theta} (1 + \bar{\theta})^{-2}$ and $\bar{\theta} = \left(\frac{x_2}{x_1}\right)^{\alpha_x} \left(\frac{d_1}{d_2}\right)^{\alpha_y}$. Taking the derivative of y_i^* with respect to players' long-run efforts x_1 and x_2 , we find that $\frac{\partial y_i^*}{\partial x_1} = \frac{\alpha_y v \alpha_x \bar{\theta}}{d_i x_1} \frac{\bar{\theta} - 1}{(1 + \bar{\theta})^3}$ and $\frac{\partial y_i^*}{\partial x_2} = \frac{\alpha_y v \alpha_x \bar{\theta}}{d_i x_2} \frac{1 - \bar{\theta}}{(1 + \bar{\theta})^3}$ for $i = 1, 2$. It follows that $\text{sign}\left(\frac{\partial y_i^*}{\partial x_1}\right) = \text{sign}(\bar{\theta} - 1)$ and $\text{sign}\left(\frac{\partial y_i^*}{\partial x_2}\right) = \text{sign}(1 - \bar{\theta})$. **Q.E.D.**

Proof of Proposition 2. Suppose that in the stage-2 contest the long-run efforts are fixed at the interior equilibrium levels $(x_1, x_2) = (x_1^*, x_2^*)$. Then, according to Proposition 1, there exists a unique Nash equilibrium of the stage-2 contest for $\alpha_y \leq 1$. Player i spends $y_i^* = \frac{\alpha_y v \theta^*}{d_i} (1 + \theta^*)^{-2}$ in the short-run activity, and the equilibrium cost of effort in the short-run activity is $C_y^* = 2\alpha_y v \theta^* (1 + \theta^*)^{-2}$, where $\theta^* = \left(\frac{x_2^*}{x_1^*}\right)^{\alpha_x} \left(\frac{y_2^*}{y_1^*}\right)^{\alpha_y} = \left(\frac{x_2^*}{x_1^*}\right)^{\alpha_x} \left(\frac{d_1}{d_2}\right)^{\alpha_y}$ denotes the effective relative strength of player 2 in the subgame perfect Nash equilibrium of the two-stage contest.

To characterize the equilibrium θ^* , we need to examine the first-order condition (5) for both players. Player 1's first-order condition for $x_1 > 0$ can be written as

$$\frac{\partial \tilde{\Pi}_1(x_1, x_2)}{\partial x_1} = \left(1 + \alpha_y \frac{1 - \bar{\theta}}{1 + \bar{\theta}}\right) v \frac{\alpha_x \bar{\theta}}{x_1} (1 + \bar{\theta})^{-2} - c_1 = 0, \quad (\text{A1})$$

where $\bar{\theta} = \left(\frac{x_2}{x_1}\right)^{\alpha_x} \left(\frac{d_1}{d_2}\right)^{\alpha_y}$. Hence, the equilibrium x_1^* and $\theta^* = \left(\frac{x_2^*}{x_1^*}\right)^{\alpha_x} \left(\frac{d_1}{d_2}\right)^{\alpha_y}$ must satisfy

$$c_1 x_1^* = \alpha_x v \theta^* (1 + \theta^*)^{-2} \left(1 + \alpha_y \frac{1 - \theta^*}{1 + \theta^*}\right). \quad (\text{A2})$$

Similarly, player 2's first-order condition with respect to x_2 implies

$$c_2 x_2^* = \alpha_x v \theta^* (1 + \theta^*)^{-2} \left(1 + \alpha_y \frac{\theta^* - 1}{1 + \theta^*}\right). \quad (\text{A3})$$

It follows that

$$\frac{x_2^*}{x_1^*} = \frac{c_1 (1 - \alpha_y + (1 + \alpha_y)\theta^*)}{c_2 (1 + \alpha_y + (1 - \alpha_y)\theta^*)} \quad (\text{A4})$$

and that the total cost of long-run efforts is $C_x^* = c_1 x_1^* + c_2 x_2^* = 2\alpha_x v \theta^* (1 + \theta^*)^{-2}$. Combining the results for the equilibrium cost of effort in short-run and long-run activities, we find that the total expenditures in the two-stage contest are $C^* = 2\alpha v \theta^* (1 + \theta^*)^{-2}$, where $\alpha = \alpha_x + \alpha_y$.

Substituting (A4) into $\theta^* = \left(\frac{x_2^*}{x_1^*}\right)^{\alpha_x} \left(\frac{d_1}{d_2}\right)^{\alpha_y}$ yields $\theta^* = \left(\frac{1 - \alpha_y + (1 + \alpha_y)\theta^*}{1 + \alpha_y + (1 - \alpha_y)\theta^*}\right)^{\alpha_x} \theta$. Therefore, the equilibrium θ^* satisfies $\varphi(\theta^*) = 0$, where $\varphi(r) \equiv \left(\frac{r}{\theta}\right)^{1/\alpha_x} - \frac{1 - \alpha_y + (1 + \alpha_y)r}{1 + \alpha_y + (1 - \alpha_y)r}$. Next, we prove that for $\alpha_y < 1$ equation $\varphi(r) = 0$ has a positive root θ^* , and it is farther away from one than θ for $\theta \neq 1$. The function $\varphi(r)$ is continuous on $[0, \infty)$ and $\varphi(0) = -\frac{1 - \alpha_y}{1 + \alpha_y} < 0$. Suppose first that $\theta < 1$. Then, $\varphi(\theta) = \frac{2\alpha_y(1 - \theta)}{1 + \alpha_y + (1 - \alpha_y)\theta} > 0$. Hence, by the Intermediate Value Theorem, there exists $\theta^* \in (0, \theta)$ such that $\varphi(\theta^*) = 0$. Next, suppose $\theta > 1$. Then, $\varphi(\theta) < 0$, and there exists $r > \theta$ such that $\varphi(r) > 0$. For example, for $r = \theta \left(\frac{1 + \alpha_y}{1 - \alpha_y}\right)^{\alpha_x} > \theta$, $\varphi(r) = \frac{1 + \alpha_y}{1 - \alpha_y} - \frac{1 - \alpha_y + (1 + \alpha_y)r}{1 + \alpha_y + (1 - \alpha_y)r} > 0$. Hence, there exists $\theta^* \in \left(\theta, \theta \left(\frac{1 + \alpha_y}{1 - \alpha_y}\right)^{\alpha_x}\right)$ such that $\varphi(\theta^*) = 0$. Lastly, if $\theta = 1$, then $\theta^* = 1$ is a solution to $\varphi(\theta^*) = 0$. We have shown that for $\alpha_y < 1$, $\varphi(r) = 0$ has a positive root θ^* ; if $\theta < 1$, then $\theta^* < \theta < 1$; if $\theta > 1$, then $\theta^* > \theta > 1$;

and if $\theta = 1$, then $\theta^* = \theta = 1$.⁵ For $\alpha_y = 1$, we can directly check that a positive root $\theta^* = \theta^{\frac{1}{1-\alpha_x}}$ exists for $\alpha_x \in (0, 1)$, and the two-stage contest is effectively more unbalanced than the one-stage contest, provided $\theta \neq 1$. For $\alpha_x = \alpha_y = 1$, the solution exists only when $\theta = 1$, and in that case $\theta^* = \theta$.

To check that the solution to $\varphi(\theta^*) = 0$ together with (A2) and (A3) indeed characterize the equilibrium efforts of the players, we need to examine the second-order conditions. Defining function $g(x) \equiv (1 + (1 - \alpha_y)x)(1 + x)^{-2}$, player 1's reduced payoff can be written as $\tilde{\Pi}_1(x_1, x_2) = g(\bar{\theta})v - c_1x_1$ for $x_1 > 0$. Differentiating the payoff twice with respect to the long-run effort x_1 , we obtain the second-order sufficient condition for player 1's optimization problem: $\frac{1}{v} \frac{\partial^2 \tilde{\Pi}_1(x_1, x_2)}{\partial x_1^2} = g''(\bar{\theta}) \frac{\partial \bar{\theta}}{\partial x_1} + g'(\bar{\theta}) \frac{\partial^2 \bar{\theta}}{\partial x_1^2} < 0$. For the second-order condition to hold for any $x_1 > 0$, it suffices to require that $\bar{\theta}(1 - \alpha_y) + \alpha_y + 1 \geq 0$, which holds for $\alpha_y \leq 1$. Similarly, the second-order condition holds for player 2 if $\alpha_x \leq 1$. It follows that players' reduced payoffs are strictly concave in players' own long-run efforts for $\alpha_y \leq 1$. Hence, the solution to the first-order conditions maximizes players' payoffs, and the equilibrium is unique.

Finally, to ensure the existence of an interior equilibrium, we need to show that both players' equilibrium payoffs are nonnegative. Player 1's equilibrium payoff in the two-stage contest is $\Pi_1^* = (1 + \theta^*)^{-1} v - \alpha v \theta^* (1 + \theta^*)^{-2}$. Then, $\Pi_1^* \geq 0$ if and only if $1 + \theta^* (1 - \alpha) \geq 0$. This holds for $\alpha \leq 1$ or for $\alpha > 1$ and $\theta^* \leq \frac{1}{\alpha - 1}$. Similarly, $\Pi_2^* \geq 0$ for $\alpha \leq 1$ or for $\alpha > 1$ and $\theta^* \geq \alpha - 1$. Hence, for $\alpha \leq 1$ and for $\alpha \in (1, 2]$ and $\theta^* \in [\alpha - 1, \frac{1}{\alpha - 1}]$, equilibrium payoffs are nonnegative. For $\alpha > 1$, $\alpha - 1 \leq \frac{1}{\alpha - 1}$ if and only if $\alpha \leq 2$. Hence, there is no pure-strategy equilibrium for $\alpha > 2$.⁶

All that is left to show is that for $\alpha_x \leq 1$ and $\alpha_y \leq 1$, $\theta^* \in [\alpha - 1, \frac{1}{\alpha - 1}]$ if $\theta \in$

⁵ When $\alpha_x = 1$ and $\alpha_y \in (0, 1)$, equation (7) can be written as $\theta^{*2} + b\theta^* - \theta = 0$ with $b = \frac{(1+\alpha_y)(1-\theta)}{(1-\alpha_y)}$, and its unique positive solution is $\theta^* = \frac{1}{2}(\sqrt{4\theta + b^2} - b)$.

⁶ Similarly, Baye et al. (1994) find that there does not exist a pure-strategy Nash equilibrium in the standard Tullock one-activity contest model if the discriminatory power of the contest is greater than two.

$[(\alpha - 1)^{1-\alpha_x}, (\alpha - 1)^{\alpha_x-1}]$. For $\alpha_x = 1$ and $\alpha_y \in (0, 1]$ this holds because $\theta^* = \theta \in [\alpha - 1, \frac{1}{\alpha-1}] = [\alpha_y, \frac{1}{\alpha_y}]$ if $\theta = 1$. Consider the case $\alpha_x < 1$. Totally differentiating $\varphi(\theta^*(\alpha_x, \alpha_y, \theta); \alpha_x, \alpha_y, \theta) = 0$ with respect to α_y , we find that $\frac{\partial \theta^*}{\partial \alpha_y} = -\frac{\partial \varphi}{\partial \alpha_y} / \frac{\partial \varphi}{\partial \theta^*}$. From $\frac{\partial \varphi}{\partial \theta^*} = \frac{1}{\alpha_x \theta^*} \left(\frac{\theta^*}{\theta}\right)^{\frac{1}{\alpha_x}} - 4 \frac{\alpha_y}{(1+\alpha_y+(1-\alpha_y)\theta^*)^2}$ and $\varphi(\theta^*) = \left(\frac{\theta^*}{\theta}\right)^{\frac{1}{\alpha_x}} - \frac{1-\alpha_y+(1+\alpha_y)\theta^*}{1+\alpha_y+(1-\alpha_y)\theta^*} = 0$, it follows that $\frac{\partial \varphi}{\partial \theta^*} = \frac{(1-\alpha_y)(2(1-\alpha_y)\theta^*+(1+\alpha_y)(1+\theta^{*2}))}{\alpha_x \theta^* (1+\alpha_y+(1-\alpha_y)\theta^*)^2} > 0$. First, suppose $\theta < 1$. Then, $\theta^* < \theta < 1$ and $\frac{\partial \varphi}{\partial \alpha_y} = 2 \frac{1-\theta^{*2}}{(1+\alpha_y+(1-\alpha_y)\theta^*)^2} > 0$. This means that in the case of $\theta < 1$, $\frac{\partial \theta^*}{\partial \alpha_y} < 0$, and the lowest θ^* is the one for $\alpha_y = 1$: $\theta^* = \theta^{\frac{1}{1-\alpha_x}}$. Hence, it suffices to require $\theta^* = \theta^{\frac{1}{1-\alpha_x}} \geq \alpha - 1$, which is equivalent to $\theta \geq (\alpha - 1)^{1-\alpha_x}$. Similarly, suppose $\theta > 1$. Then, $\theta^* > \theta > 1$ and $\frac{\partial \varphi}{\partial \alpha_y} < 0$. Since $\frac{\partial \varphi}{\partial \theta^*} > 0$, it follows that $\frac{\partial \theta^*}{\partial \alpha_y} > 0$, and the highest θ^* is the one for $\alpha_y = 1$: $\theta^* = \theta^{\frac{1}{1-\alpha_x}}$. Hence, it suffices to require $\theta^* = \theta^{\frac{1}{1-\alpha_x}} \leq \frac{1}{\alpha-1}$, which is equivalent to $\theta \leq (\alpha - 1)^{\alpha_x-1}$. We therefore conclude that there exists a unique, interior equilibrium if $\alpha \leq 1$ or if $\alpha \in (1, 2]$, $\alpha_x \leq 1$, $\alpha_y \leq 1$, and $\theta \in [(\alpha - 1)^{1-\alpha_x}, (\alpha - 1)^{\alpha_x-1}]$. **Q.E.D.**

Proof of Proposition 3. Rent dissipation in the one-stage contest is $D^{*1} = 2\alpha\theta(1+\theta)^{-2}$, where $\theta = \left(\frac{c_1}{c_2}\right)^{\alpha_x} \left(\frac{d_1}{d_2}\right)^{\alpha_y}$. According to Proposition 2, rent dissipation is $D^* = 2\alpha\theta^*(1+\theta^*)^{-2}$ in the subgame perfect equilibrium of the two-stage contest, where $\theta^* \equiv \left(\frac{x_2^*}{x_1^*}\right)^{\alpha_x} \left(\frac{d_1}{d_2}\right)^{\alpha_y}$, and θ^* is farther away from 1 than θ as long as $\theta \neq 1$. Since the function $f(x) = x(1+x)^{-2}$ is continuous for $x > 0$, strictly increasing for $x \in (0, 1)$, and strictly decreasing for $x > 1$, rent dissipation is lower in the two-stage than in the one-stage contest. **Q.E.D.**

Appendix B: Two-Stage K -Activity Contests

Let $\mathbf{K} \equiv \{1, \dots, K\}$ be the set of all activities. A subset \mathbf{K}_x of activities are chosen at stage 1 (the long run), while the remaining activities $k \in \mathbf{K}_y$ are chosen at stage 2 (the short run). Player i 's payoff in the K -activity contest with constant marginal costs of effort $c_{ik} > 0$ is: $\Pi_i(\mathbf{x}_1, \mathbf{x}_2) = \frac{f(\mathbf{x}_i)}{f(\mathbf{x}_1) + f(\mathbf{x}_2)} v - \sum_{k \in \mathbf{K}} c_{ik} x_{ik}$, where $f(\mathbf{x}_i) = \prod_{k \in \mathbf{K}} x_{ik}^{\alpha_k}$, $\mathbf{x}_i = (x_{i1}, \dots, x_{iK})$

is the vector of player i 's efforts $x_{ik} \geq 0$ in each activity $k \in \mathbf{K}$, $\alpha_k > 0$, $i = 1, 2$. For the K -activity contest, we redefine $\alpha = \sum_{k \in \mathbf{K}} \alpha_k$, $\alpha_x = \sum_{k \in \mathbf{K}_x} \alpha_k$, and $\alpha_y = \sum_{k \in \mathbf{K}_y} \alpha_k$ to be the overall, long-run, and short-run discriminatory power of the contest.

Using $x_i = \prod_{k \in \mathbf{K}_x} x_{ik}^{\alpha_k / \alpha_x}$ and $y_i = \prod_{k \in \mathbf{K}_y} x_{ik}^{\alpha_k / \alpha_y}$, player i 's contest success function can be written as in equation (2) for the two-activity contest. Player i 's cost function is $C_i(x_i^{\alpha_x}, y_i^{\alpha_y}) \equiv \min \sum_{k \in \mathbf{K}} c_{ik} x_{ik}$ s.t. $\prod_{k \in \mathbf{K}_x} x_{ik}^{\alpha_k} = x_i^{\alpha_x}$ and $\prod_{k \in \mathbf{K}_y} x_{ik}^{\alpha_k} = y_i^{\alpha_y}$. Using the Lagrangian multiplier method, we find that player i spends effort in activities k and l to the extent that the marginal rate of technical substitution between the efforts is equal to the ratio of their costs: $\frac{\alpha_k}{\alpha_l} \frac{x_{il}}{x_{ik}} = \frac{c_{lk}}{c_{il}}$ for $k \neq l \in \mathbf{K}_x$ and for $k \neq l \in \mathbf{K}_y$, and $C_i(x_i^{\alpha_x}, y_i^{\alpha_y}) = c_i x_i + d_i y_i$, where $c_i = \alpha_x \prod_{k \in \mathbf{K}_x} \left(\frac{c_{ik}}{\alpha_k} \right)^{\alpha_k / \alpha_x}$, and $d_i = \alpha_y \prod_{k \in \mathbf{K}_y} \left(\frac{c_{ik}}{\alpha_k} \right)^{\alpha_k / \alpha_y}$. We conclude that the K -activity contest reduces to the two-activity contest with payoffs as in equation (2).

Proposition 1 describes the equilibrium in stage 2 of the two-stage two-activity contest as dependent on $\bar{\theta} = \left(\frac{x_2}{x_1} \right)^{\alpha_x} \left(\frac{d_1}{d_2} \right)^{\alpha_y} = \prod_{k \in \mathbf{K}_x} \left(\frac{x_{2k}}{x_{1k}} \right)^{\alpha_k} \prod_{k \in \mathbf{K}_y} \left(\frac{c_{1k}}{c_{2k}} \right)^{\alpha_k}$. Hence, the equilibrium in the short-run K -activity contest exists under the same conditions and player i 's equilibrium effort and the total cost of effort in short-run activities are $x_{ik}^* = \frac{\alpha_k}{c_{ik}} \frac{d_i}{\alpha_y} y_i^* = \frac{\alpha_k}{c_{ik}} v \bar{\theta} (1 + \bar{\theta})^{-2}$ and $C_y^* = \sum_{k \in \mathbf{K}_y} c_{ik} x_{ik}^* = 2\alpha_y v \bar{\theta} (1 + \bar{\theta})^{-2}$. Corollary 1 extends to the two-stage K -activity contest because $\text{sign}\left(\frac{dx_{ik}^*}{dx_{1j}}\right) = \text{sign}(\bar{\theta} - 1)$ and $\text{sign}\left(\frac{dx_{ik}^*}{dx_{2j}}\right) = \text{sign}(1 - \bar{\theta})$ for $k \in \mathbf{K}_y$ and $j \in \mathbf{K}_x$. Using Propositions 2 and 3 for the two-stage two-activity contest and the fact that $\theta \equiv \left(\frac{c_1}{c_2} \right)^{\alpha_x} \left(\frac{d_1}{d_2} \right)^{\alpha_y} = \prod_{k \in \mathbf{K}} \left(\frac{c_{1k}}{c_{2k}} \right)^{\alpha_k}$ and $\theta^* \equiv \left(\frac{x_2^*}{x_1^*} \right)^{\alpha_x} \left(\frac{d_1}{d_2} \right)^{\alpha_y} = \prod_{k \in \mathbf{K}} \left(\frac{x_{2k}^*}{x_{1k}^*} \right)^{\alpha_k}$, we obtain the same results for the two-stage K -activity contest.

REFERENCES

- Amegashie, J. Atsu (2004). "Burning Out in Sequential Elimination Contests," University of Guelph, Working Paper.
- Amegashie, J. Atsu and Runkel, Marco (2007). "Sabotaging Potential Rivals," *Social Choice and Welfare* 28, 143-162.

- Arbatskaya, Maria and Mialon, Hugo M. (2010). "Multi-Activity Contests," *Economic Theory* 43, 23-43.
- Bagwell, Kyle (2007). "The Economic Analysis of Advertising," in Armstrong and Porter, eds, *Handbook of Industrial Organization, Volume 3*, 1701-1844, Elsevier.
- Baik, Kyung Hwan and Shogren, Jason F. (1992). "Strategic Behavior in Contests: Comment," *American Economic Review* 83, 359-362.
- Baik, Kyung Hwan and Lee, Sanghack (2000). "Two-stage Rent-Seeking Contests with Carryovers," *Public Choice* 103, 285-296.
- Baye, Michael R., Kovenock, Dan and de Vries, Casper G. (1994). "The Solution to the Tullock Rent-Seeking Game when $R > 2$: Mixed-Strategy Equilibria and Mean Dissipation Rates," *Public Choice* 81, 363-380.
- Caves, Richard E., Greene, David P. (1996). "Brands' Quality Levels, Prices, and Advertising outlays: Empirical Evidence on Signals and Information Costs," *International Journal of Industrial Organization* 14, 29-52.
- Chen, Kong-Pin (2003). "Sabotage in Promotion Tournaments," *Journal of Law, Economics, and Organization* 19, 119-140.
- Clark, Derek J. and Konrad, Kai A. (2007). "Contests with Multi-Tasking," *Scandinavian Journal of Economics* 109, 303-319.
- Cunha, Flavio and Heckman, James J. (2007). "The Technology of Skill Formation," *American Economic Review* 97, 31-47.
- Cunha, Flavio, Heckman, James J., and Schennach, Susanne (2010). "Estimating the Technology of Cognitive and Noncognitive Skill Formation," *Econometrica*, In Press.
- Dixit, Avinash K. (1987). "Strategic Behavior in Contests," *American Economic Review* 77, 891-898.
- Dixit, Avinash K. (1999). "Strategic Behavior in Contests: Reply," *American Economic Review* 89, 694.
- Epstein, Gil S. and Hefeker, Carsten (2003). "Lobbying Contests with Alternative Instruments," *Economics of Governance* 4, 81-89.
- Fu, Qiang and Lu, Jingfeng (2010). "The Optimal Multi-Stage Contest," *Economic Theory*, In Press.
- Gradstein, Mark and Konrad, Kai A (1999). "Orchestrating Rent Seeking Contests," *Economic Journal* 109, 536-545.
- Harbaugh, Rick and Klumpp, Tilman (2005). "Early Round Upsets and Championship Blowouts," *Economic Inquiry* 43, 316-329.
- Konrad, Kai A. and Kovenock, Dan (2009). "Multi-battle Contests," *Games and Economic Behavior* 66, 256-274.

- Konrad, Kai A. (2000). "Sabotage in Rent-Seeking Contests," *Journal of Law, Economics, and Organization* 16, 155-165.
- Kräkel, Matthias (2004). "R&D Spillovers and Strategic Delegation in Oligopolistic Contests," *Managerial and Decision Economics* 25, 147-156.
- Kräkel, Matthias (2005). "Helping and Sabotaging in Tournaments," *International Game Theory Review* 7, 211-228.
- Lee, Sanghack (2003). "Two-Stage Contests With Additive Carryovers," *International Economic Journal* 17, 83-99.
- Münster, Johannes (2007). "Contests with Investment," *Managerial and Decision Economics* 28, 849-862.
- Qiu, Larry D. (1997). "On the Dynamic Efficiency of Bertrand and Cournot Equilibrium," *Journal of Economic Theory* 75, 213-229.
- Schmitt, Pamela, Shupp, Robert, Swope, Kurtis, and Cadigan, John (2004). "Multi-Period Rent-Seeking Contests with Carryover: Theory and Experimental Evidence," *Economics of Governance* 5, 187-211.
- Sheremeta, Roman M. (2010). "Experimental Comparison of Multi-Stage and One-Stage Contests," *Games and Economic Behavior* 68, 731-747.
- Soubeyran, Raphaël (2009). "Contest with Attack and Defense: Does Negative Campaigning Increase or Decrease Voter Turnout?" *Social Choice and Welfare* 32, 337-353.